A Simple Cooperative Diversity Method Based on Network Path Selection

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Abstract

Cooperative diversity has been recently shown to provide dramatic gains in slow fading wireless environments. However most of the proposed solutions require distributed space-time protocols, many of which are infeasible if there is more than one cooperative relay. We propose a novel scheme, ”opportunistic relaying” that alleviates these problems and provides diversity gains on the order of the number of relays in the network. Our scheme first selects the best relay from a set of M available relays and then uses this ”best” relay for cooperation between the source and the destination. We develop and analyze a distributed method to select the best relay based on the local measurements of the channel conditions by the relays. This method also requires no explicit communication among the relays. The success (or failure) to select the best available path depends on the statistics of the wireless channel, and a methodology to evaluate performance for any kind of wireless channel statistics, is provided. Information theoretic analysis of outage probability shows that our scheme achieves the same diversity-multiplexing tradeoff as achieved by more complex protocols, where coordination and distributed space-time coding for M nodes is required, such as those proposed in [8]. The simplicity of the technique, allows for immediate implementation in existing radio hardware and its adoption could provide for improved flexibility, reliability and efficiency in future 4G wireless systems.

I. Introduction

In this work, we propose and analyze *Opportunistic Relaying*, which is a novel method to select the ”best” end-to-end path between a source and destination of wireless information. The setup includes a set of cooperating relays which are willing to forward received information towards the destination and opportunistic relaying is about a distributed algorithm that selects the most appropriate relay to forward information towards the receiver. The decision is based on the end-to-end instantaneous wireless channel conditions and the algorithm is distributed among the cooperating wireless terminals.

The best relay selection algorithm lends itself naturally into cooperative diversity protocols, which have been recently proposed to improve reliability in wireless communication systems using distributed virtual antennas. The key idea behind these protocols is to create additional paths between the source and destination terminals using intermediate relay nodes. Reliable communication between the source and destination is possible if *any* one of the paths is strong enough. Several cooperative protocols were proposed in [7] and their performance was measured in terms of the diversity-multiplexing tradeoff [12]. Their basic setup included one sender, one receiver and one intermediate relay node and both analog as well as digital processing at the relay node were considered. Subsequently the case when there are several intermediate relay nodes was considered in [1], [8] and it was shown that the diversity gains achieved are on the order of the number of relay nodes in the network.

Unfortunately, these protocols require the implementation of distributed space time codes across the relay nodes and this is practically not feasible: while well known space time codes such as the Alamouti scheme, can be implemented in the single relay case, to the best of our knowledge, the current state of art in space time codes is far from developing practical schemes when there is more than one intermediate relay [1]. Apart from practical space-time coding, the formation of virtual antenna arrays using individual terminals distributed in space, requires significant amount of coordination, which is still an open research problem. Specifically, the formation of cooperating groups of terminals involves distributed algorithms [8] while synchronization at the packet level is required among several different transmitters. Those additional requirements for cooperative diversity demand significant modifications to almost all layers of the communication stack (up to the routing layer) which has been built according to ”traditional”, point-to-point (non-cooperative) communication.

In such distributed settings, opportunistic relaying algorithm provides a practical alternative to select simply the best available relay between the source and the destination rather than involving all available relays and then using the well known space time protocols used in the single relay case. Additionally, the algorithm itself provides for the necessary coordination in time and group formation among the cooperating terminals.

A natural question to consider is how much performance loss is incurred through this practically appealing alternative as compared to the more complicated protocols. In this work we study this performance loss. Surprisingly we observe that in terms of the diversity multiplexing tradeoff, there is no performance loss

[1]At least we are not aware of generalizations of Alamouti scheme that achieve the entire diversity multiplexing tradeoff curve.

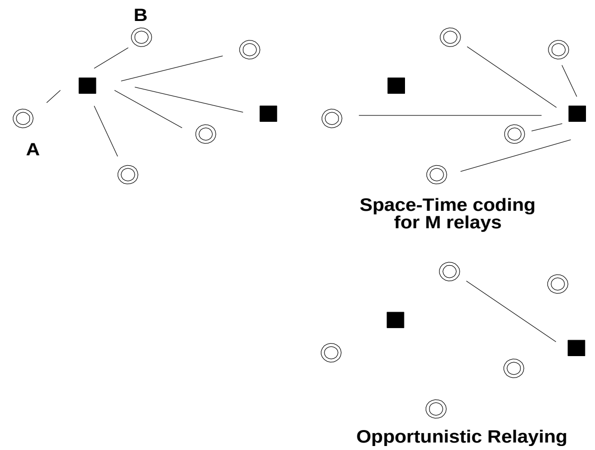

Fig. 1. A transmission is overheard by neighboring nodes. Practical cooperative diversity schemes should address which nodes are beneficial to relay information to the receiver. Distributed Space-Time coding is needed so that all overhearing nodes could simultaneously transmit. In this work we analyze "Opportunistic Relaying" where the relay with the strongest transmitter-relay-receiver channel is selected, among several candidates, in a distributed fashion.

compared to more complicated protocols, such as that proposed in [8]. Intuitively, the gains in cooperative diversity do not come from using complex schemes, but rather from the fact that we have enough relays in the system to provide sufficient diversity.

In the following sections, we first explain in detail the motivation behind this work. We then describe the algorithm and present a simple probabilistic analysis that quantifies the success (or failure) of selecting the most appropriate path. Given the dependence of the selection scheme on the wireless channel conditions, performance is evaluated according to the statistics of the wireless channel. The analytical solution applies for any kind of wireless channel distribution and specific examples on Rayleigh and Ricean fading are given. We continue with the diversity-multiplexing tradeoff analysis of the proposed scheme. We conclude this work in the last section.

The simplicity of the technique, allows for immediate implementation in existing radio hardware. An implementation of the scheme using custom radio hardware is reported in [3], [4]. Its adoption could provide for improved flexibility (since the technique addresses coordination issues), reliability and efficiency (since the technique inherently builds upon diversity) in future 4G wireless systems.

II. Motivation

Cooperative diversity in its simplest form of a transmitter, single relay and receiver, involves relaying of information from a neighboring overhearing node rather than having the initial transmitter repeat its transmission. The receiver would exploit both direct and relayed transmission over two statistically independent paths and therefore resistance to fading would improve because of diversity.

In figure 1 a transmitter transmits its information towards the receiver while all the neighboring nodes are in listening mode. For a practical cooperative diversity in a three-node setup, the transmitter should know that allowing a relay at location (B) to relay information, would be more efficient than repetition from the transmitter itself. This is not a trivial task and such event depends on the wireless channel conditions between transmitter and receiver as well as between transmitter-relay and relay-receiver. What if the relay is located in position (A)? And how could cooperative diversity scale in practice when larger number of relays are used?

Current proposals allow for all overhearing nodes to relay simultaneously during the second step of the scheme (figure 1). Opportunistic relaying needs only two transmissions, one from source and one from ”best” relay. Therefore, opportunistic relaying simplifies and address cooperative communication as a Routing problem: how could the best end-to-end path be selected in a distributed and dynamic way that adapts to the wireless channel conditions? This perspective motivated our work and allowed the implementation of a demonstration on cooperative diversity, using opportunistic relaying and custom radio hardware [3], [4].

MIMO theory suggests that selection diversity in a multi-antenna transceiver (i.e. selecting the antenna with the highest SNR among M antennas) provides for a diversity gain of M, even though one antenna is used [9]. Therefore, it was natural to explore ”virtual” antenna arrays with the same behavior. Opportunistic relaying is based on similar ideas and provides for diversity gain equal to the number of cooperating nodes, even though only two nodes transmit.

III. DESCRIPTION OF OPPORTUNISTIC RELAYING

According to opportunistic relaying, a single relay among a set of M nodes is selected, depending on which relay provides for the ”best” end-to-end path between source and destination (figure 1, 2). The wireless channel a_{si} between source and each relay i, as well as the channel a_{id} between relay i and destination affect performance. Since communication among all relays should be minimized for reduced overall overhead, a method based on time was selected: each relay starts a timer from a parameter h_i based on the channel conditions a_{si}, a_{id}. The timer of the relay with the best end-to-end channel conditions will expire first. All relays, while waiting for their timer to reduce to zero (i.e. to expire) are in listening mode. As soon as they hear another relay to forward information (the best relay), they back off.

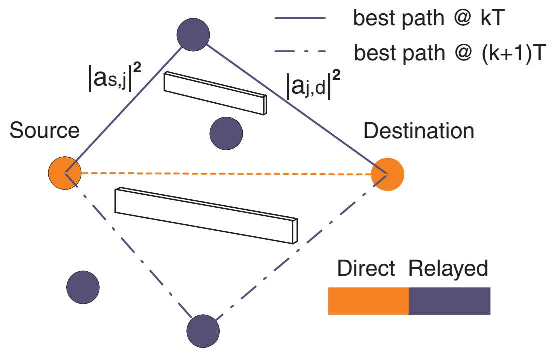

Fig. 2. Source transmits to destination and neighboring nodes overhear the communication. The "best" relay among M candidates is selected to relay information, via a distributed mechanism and based on instantaneous end-to-end channel conditions.

For the case where all relays can listen source and destination, but they are "hidden" from each other (i.e. they can not listen each other), the best relay could notify the destination with a short duration *flag* packet and the destination could then notify all relays with a short broadcast message.

All the above assume that all relays start their timers at the same time. Synchronization can be easily achieved by the exchange of Ready-to-Send (RTS) and Clear-to-Send (CTS) packets between source and destination. Relays can start their timers as soon as they receive the CTS packet. In that case, synchronization error on the order of propagation delay differences across all destination-relay pairs should be taken into account. For the cases where source and destination are not in direct range, they need to synchronize their RTS/CTS exchange by other means. For example, Network Time Keeping algorithms in client/server setups, such as those examined in [2] could be employed. Or fully decentralized solutions for network time keeping could be facilitated, such as those demonstrated in [5].

The RTS/CTS mechanism, existent in most MAC protocols, is also necessary for channel estimation at the relays: the transmission of RTS from the source allows for the estimation of the wireless channel a_{si} between source and relay i, at each relay i. Similarly, the transmission of CTS from the destination, allows for the estimation of the wireless channel a_{id} between relay i and destination, at each relay i, according to the reciprocity theorem.

The channel estimates a_{si}, a_{id} at each relay, describe the quality of the wireless path source-relay-destination, for each relay i. Opportunistic relaying is about selecting the "best" path among M possible options. Since the two hops are equally important for end-to-end performance, each relay should quantify its appropriateness as an active relay, using a function that balances the two hops. Two functions are used in this work. Under policy I, the minimum of the two is selected (equation (1)), while under policy II, the harmonic mean of the two is used (equation (2)).

Under policy I:

$$h_i = \min\{|a_{si}|^2, |a_{id}|^2\} \tag{1}$$

Under policy II:

$$h_i = \frac{2}{\frac{1}{|a_{si}|^2} + \frac{1}{|a_{id}|^2}} = \frac{2\ |a_{si}|^2\ |a_{id}|^2}{|a_{si}|^2 + |a_{id}|^2} \tag{2}$$

The relay i that maximizes function h_i is the one with the ”best” end-to-end path between initial source and final destination (equation (4)). All relays will start their timer with an initial value, inversely proportional to the end-to-end channel quality h_i, according to the following equation:

$$X_i = \frac{\lambda}{h_i} \tag{3}$$

λ is a constant that converts units of channel quality $|a|^2$ in units of time. It has units *”unit of time” x ”unit of* $|a|^2$*”*. For example, if time is measured in $\mu secs$ and $|a|^2$ in units of power, then λ could have values in $\mu sec\ \mu Watts$.

Assuming synchronization among the relays (either from the RTS/CTS exchange between source and destination or through explicit multi-hop schemes as explained above), all relays start their timer simultaneously, with different initial values, depending on their channel realizations. The ”best” relay, is the one with its timer reduced to zero first (since it started from a smaller initial value, according to equations (5), (3). This is the relay b that will participate in forwarding information. The rest of the relays, will back off.

$$h_b = \max\{h_i\}, \iff \tag{4}$$

$$X_b = \min\{X_i\},\ i \in [1..M]. \tag{5}$$

In the following section, we quantify the probability any other relay’s timer (apart from the ”best” relay) expires within a time interval c, from the instant when the best relay timer expired. In that way, we are able to calculate the probability this scheme to succeed in selecting the ”best” relay or fail when two or more relay transmissions collide.

As can be seen from the above equations, the scheme depends on the instantaneous channel realizations or equivalently, on received SNRs. Therefore, the best relay selection algorithm should be applied as often as the wireless channel changes and not as often the source transmits information. That rate of wireless channel change depends on the *coherence time* of the channel. The advantage of opportunistic relaying is that it requires no explicit communication among the relays.

In the following section we will calculate the probability of successful relay selection, even at the case where the relays are hidden from each other.

IV. Probabilistic Analysis of Opportunistic Relaying

The probability of having two or more relay timers expire ”at the same time” is zero. However, the probability of having two or more relay timers expire within the same time interval c is non zero and can be analytically evaluated, given knowledge of the wireless channel statistics.

The only case where opportunistic relay selections fails, is when the relays cannot listen each other and therefore, one relay can not detect that another relay is more appropriate for information forwarding. Note that we have already assumed that all relays can listen initial source and destination, otherwise they do not participate in the scheme. We will assume two extreme cases: a) all relays can listen to each other b) all relays are hidden from each other (but they can listen source and destination). In the second case, the best relay sends a flag packet to destination (or source) to notify for its candidacy, as the best relay. Then the destination (or source) notifies all relay nodes with a short broadcast message.

From figure 3, collision of two or more relays can happen if the best relay timer X_b and one or more other relays expire within $[t_L,\ t_C]$ for case (a), or within $[t_L,\ t_H]$ for case (b). In any case, the collision probability can be upper bounded by the following expression:

$$\begin{aligned} Pr(Collision) &\leq Pr(any\ X_j < X_b + c \mid j \neq b) \\ where\ X_b &= \min\{X_j\},\ j \in [1, M]\ and\ c > 0. \end{aligned} \tag{6}$$

and

(a) No Hidden Relays:

$$c = r_{max} + |v_b - v_j|_{max} + d_s \tag{7}$$

(b) Hidden Relays:

$$c = r_{max} + 2|v_b - v_j|_{max} + d_s + dur \tag{8}$$

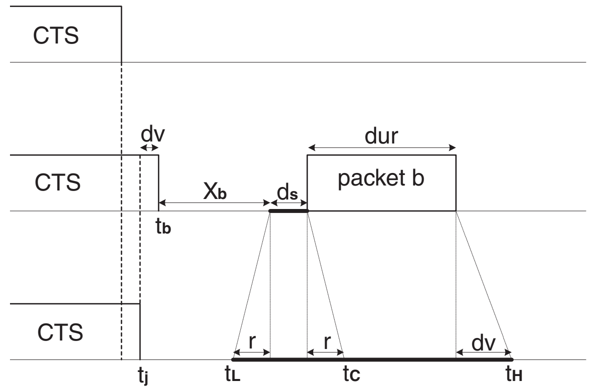

Fig. 3. The middle raw corresponds to the ”best” relay. Other relays (top or bottom raw) could erroneously be selected as ”best” relays, if their timer expired within intervals when they can not hear the best relay transmission. That can happen in the interval $[t_L, t_C]$ for case (a) (No Hidden Relays) or $[t_L, t_H]$ for case (b) (Hidden Relays). t_b, t_j are time points where reception of the CTS packet is completed at best relay b and relay j respectively.

- v_j: propagation delay between relay j and destination.
- r_{max}: maximum propagation delay between two relays.
- d_s: receive-to-transmit switch time of each radio.
- dur: duration of flag packet, transmitted by ”best” relay.

The upper bound of (6) and equations (7), (8) can be easily derived taking into account propagation delays, radio switch time and flag packet duration.

In the following section, we will provide an analytic way to calculate the upper bound of equation (6). But before doing so, we will easily show that this probability can be made arbitrary small, close to zero.

If $X_b = \min\{X_j\}, j \in [1, M]$ and $Y_1 < Y_2 < \ldots < Y_M$ the ordered random variables $\{X_j\}$ with $X_b \equiv Y_1$, then:

$$Pr(any\ X_j < X_b + c \mid j \neq b) = Pr(Y_2 < Y_1 + c) \tag{9}$$

Given that $Y_j = \lambda/h_{(j)}$, $Y_1 < Y_2 < \ldots < Y_M$ is equivalent to $1/h_{(1)} < 1/h_{(2)} < \ldots < 1/h_{(M)}$, equation (9) is equivalent to

$$Pr(Y_2 < Y_1 + c) = Pr(\frac{1}{h_{(2)}} < \frac{1}{h_{(1)}} + \frac{c}{\lambda}) \tag{10}$$

and $Y_1 < Y_2 < \ldots < Y_M \Leftrightarrow h_{(1)} > h_{(2)} \ldots > h_{(M)}$ (h, λ, c are positive numbers).

From the last equation (10), it is obvious that increasing λ at each relay (in equation (3)), reduces the probability of collision to zero since the upper bound of (10) goes to zero with increasing λ.

In practice, λ can not be made arbitrarily large, since it also ”regulates” the expected time, needed for the network to find out the ”best” relay. From equation (3) and Jensen's inequality we can see that

$$E[X_j] = E[\lambda/h_j] \geq \lambda/E[h_j] \tag{11}$$

or in other words, the expected time needed for each relay to flag its presence, is lower bounded by λ. Therefore, there is a trade-off between probability of collision and speed of relay selection. We need to have λ as big as possible to reduce collision probability and at the same time, as small as possible, to quickly select the best relay, before the channel changes again (i.e. within the coherence time of the channel).

In the following section we provide a method to quantify performance for any kind of wireless channel statistics and any kind of values for c and λ and show that the scheme can perform reasonably well.

A. Calculating $Pr(Y_2 < Y_1 + c)$

Theorem 1: The joint probability density function of the minimum and second minimum among M i.i.d. positive random variables $X_1,\ X_2, \ldots,\ X_M$, each with probability density function $f(x)$ and cumulative distribution function $F(x)$, is given by the following equation:

$$\begin{aligned} f_{Y_1,Y_2}(y_1, y_2) &= M\ (M-1)\ f(y_1)\ f(y_2)\ [1 - F(y_2)]^{M-2} \\ &\quad 0 < y_1 < y_2, \\ f_{Y_1\ Y_2}(y_1, y_2) &= 0 \\ &\quad elsewhere. \end{aligned} \tag{12}$$

where $Y_1 < Y_2 < Y_3 \ldots < Y_M$ are the M ordered random variables $X_1,\ X_2, \ldots,\ X_M$.

Proof:

Please refer to appendix I. ■

Using Theorem 1, we can prove the following lemma:

Lemma 1: Given M i.i.d. positive random variables $X_1,\ X_2, \ldots,\ X_M$, each with probability density function $f(x)$ and cumulative distribution function $F(x)$, and $Y_1 < Y_2 < Y_3 \ldots < Y_M$ the M ordered random variables $X_1,\ X_2, \ldots,\ X_M$, then $Pr(\ Y_2 < Y_1 + c)$, where $c > 0$, is given by the following equations:

$$Pr(Y_2 < Y_1 + c) = 1 - I_c \tag{13}$$

$$I_c = M\ (M-1)\ \int_c^{+\infty} f(y)\ [1-F(y)]^{M-2}\ F(y-c)\ dy \tag{14}$$

Proof:

Please refer to appendix I. ■

B. Results

Using Theorem 1 and Lemma 1 of the previous section, we can quantify $Pr(Y_2 < Y_1 + c)$, for any kind of wireless channel statistics. From the above, we have restricted discussion to identically distributed wireless channel realizations. The results could be extended to the non-identically distributed case, where geometry is taken into account. We chose to restrict the discussion to the identically distributed case for simplicity and leave the non-identical (but still independent) case for future work. In the numerical results below, we have normalized $E[|a_{si}|^2] = E[|a_{id}|^2] = 1$.

In order to exploit theorem 1 and lemma 1, we first need to calculate the probability distribution of X_i for $i\ \in [1, M]$. From equation (3) it is easy to see that the cdf $F(x)$ and pdf $f(x)$ of X_i are related to the respective distributions of h_i according to the following equations:

$$F(x) \equiv\ CDF_{X_i}(x) = Pr\{X_i\ \le\ x\} = 1 - CDF_{h_i}(\frac{\lambda}{x}) \tag{15}$$

$$f(x) \equiv\ pdf_{X_i}(x) = \frac{d}{dx}F(x) = \frac{\lambda}{x^2}\ pdf_{h_i}(\frac{\lambda}{x}) \tag{16}$$

After calculating equations (15), (16), and for a given c calculated from (7) or (8), we can calculate probability of collision using equation (29).

Before proceeding to special cases, we need to observe that for a given distribution of the wireless channel, collision performance depends only on the ratio c/λ, as can be seen from equation (10), discussed earlier.

1) Rayleigh Fading: Assuming $|a_{si}|,\ |a_{jd}|$ are i.i.d according to Rayleigh distribution, for any $i,\ j \in [1, M]$, then $|a|^2$ is distributed according to an exponential distribution, with parameter β and $E[|a|^2] = 1/\beta$.

Using the fact that the minimum of two i.i.d. exponentials is again an exponential with doubled parameter, we can calculate the distributions for h_i under policy I (equation 1). For policy II (equation 2), the distributions of the harmonic mean, have been calculated analytically in [6]. Equations (15) and (16) under the above assumptions, become:

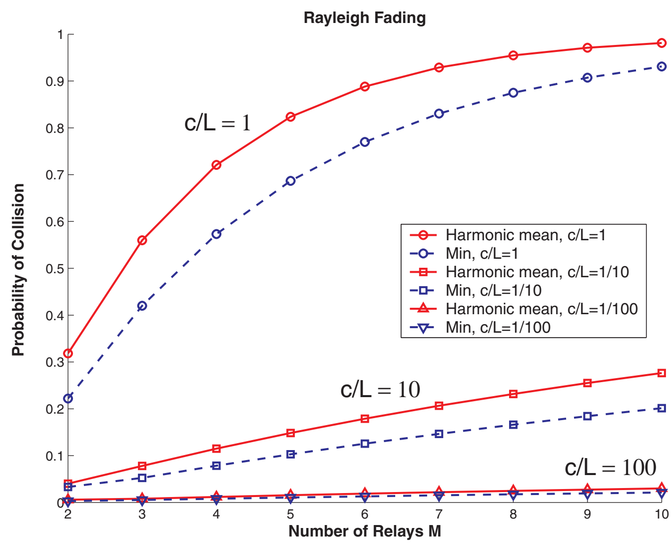

Fig. 4. Probability of collision for two policies, in Rayleigh Fading, as a function of number of relays (M). Notice that probability can be made arbitrarily small with decreased ratio c/λ.

under policy I:

$$F(x) = e^{-2\ \beta\ \lambda/x} \tag{17}$$

$$f(x) = \frac{2\ \beta\ \lambda}{x^2}\ e^{-2\ \beta\ \lambda/x} \tag{18}$$

under policy II:

$$F(x) = \frac{\lambda\ \beta}{x}\ e^{-\lambda\ \beta/x}\ K_1(\frac{\lambda\ \beta}{x}) \tag{19}$$

$$f(x) = \frac{\lambda^2}{x^3}\ b^2\ e^{-\lambda\ \beta/x}\ [K_1(\frac{\lambda\ \beta}{x}) + K_0(\frac{\lambda\ \beta}{x})] \tag{20}$$

where $K_i(x)$ is modified Bessel function of the second kind and order i.

In figure 4, equation (29) is calculated for the tho policies. We can see that the collision probability can be made arbitrarily small, with decreased ratio c/λ. That practically means that the smaller the propagation delays among relays (or equivalently the smaller the transmission range of the radios) and the faster the radios used (for smaller duration in time of the flag packet), the better performance. Practically, for $c \approx 1\ \mu sec$, corresponding to 802.11b transmission range, and average time needed for relay selection, on the order of $100\ \ \mu secs$ (or $\lambda \approx\ 100 \mu secs\ units\ of\ h_i$), the collision probability can drop below 1%.

Another interesting observation is that the two policies (harmonic mean vs minimum of the two wireless hop channel realizations) provide similar results.

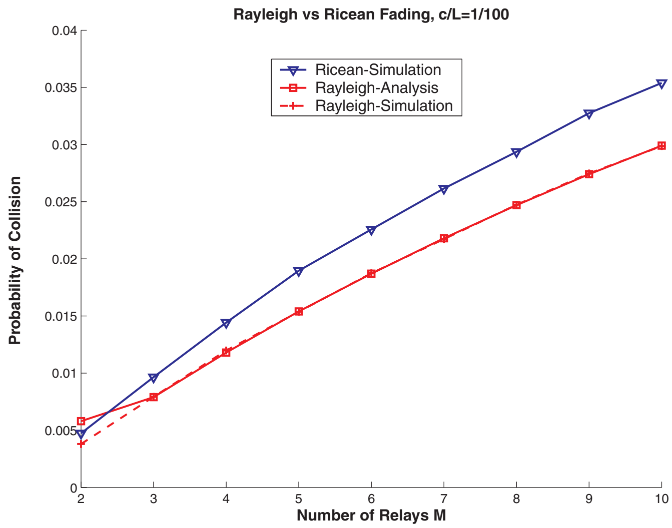

Fig. 5. Performance in Ricean and Rayleigh fading, for policy II (harmonic mean) and $c/\lambda = 1/100$.

2) Ricean Fading: It was interesting to examine the performance of opportunistic relay selection, in the case of Ricean fading, when there is a dominating path between any two communicating points, in addition to many reflecting paths and compare it to Rayleigh fading, where there is a large number of independent paths, without any dominating one.

Keeping the average value of any channel coefficient the same ($E[|a|^2] = 1$) and assuming a single dominating path and a sum of reflecting paths of equal total (sum) power, we plotted the performance of the scheme when policy II was used (the harmonic mean). The results for the Ricean case were calculated using Monte-Carlo simulation. In figure 5, the corresponding case for Rayleigh fading is also plotted, both calculated from equation (29) and from Monte-Carlo simulation. We can observe that theoretical calculation of the collision probability from equation (29) coincides with the result of Monte-Carlo simulation.

We can also see that in the Ricean case, the probability of Collision slightly increases, since now, under the i.i.d. assumption, the realizations of the wireless paths along different relays are clustered around the dominating path and vary less. On the contrary, timer expiration at the relays under Rayleigh fading varies more and therefore the interval between best relay timer and second best relay timer expiration increases.

In both cases, the scheme performs reasonably well.

V. Diversity-Multiplexing Tradeoff in Opportunistic Relaying

A. Channel Model

We consider an i.i.d slow Rayleigh fading channel model following [7]. Specifically, we assume that the transmitting node does not have any channel knowledge, while the receiving node has perfect channel knowledge. A half duplex constraint is imposed across each relay node, i.e. it cannot transmit and listen simultaneously. The opportunistic relay scenario assumes known channel gain from relay to destination, at each relay. However in this section we assume that this channel knowledge is not exploited in subsequent transmissions from each relay, to compare the performance with prior art. If the discrete time received signal at the destination and the relay node are denoted by $Y[n]$ and $Y_1[n]$, then

$$Y[n] = a_{sd}X[n] + Z[n], \quad source\ transmits \tag{21}$$

$$Y[n] = a_{rd}X_1[n] + Z[n], \quad best\ relay\ transmits$$

$$Y_1[n] = a_{sr}X[n] + Z_1[n] \tag{22}$$

Here a_{sd}, a_{rd}, a_{sr} are the respective channel gains from the source to destination, relay to destination and source to relay respectively. There are modelled as i.i.d circularly symmetric complex Gaussian $\mathcal{N}(0,1)$. The noise $Z[n]$ and $Z_1[n]$ at the destination and relay are both assumed to be i.i.d circularly symmetric complex Gaussian $\mathcal{N}(0,\sigma^2)$. $X[n]$ and $X_1[n]$ are the transmitted symbols at the transmitter and relay respectively. We impose a power constraint at both the source and the relay: $E[|X[n]|^2] \leq P$ and $E[|X_1[n]|^2] \leq P$. For simplicity, we assume that both the source and the relay to have the same power constraint. We will define $\rho \triangleq P/\sigma^2$ to be the effective signal to noise ratio (SNR) at the receivers. Note that this definition does not include the channel gain at the receivers. This setting can easily be generalized when the SNR at the relay(s) and the destination is not the same.

Under our channel modelling assumption, the channel gains between the source to relay and relay to destination are independent. We propose the following rule for selecting the best relay, according to Policy I, outlined in equation 1. We note that this choice is optimal in the sense that it incurs no loss in the diversity multiplexing tradeoff performance of several protocols we study in the subsequent section.

Rule 1: Among all the available relays, denote the relay with the largest value of $\min\{|a_{sr}|^2, |a_{rd}|^2\}$ as the best relay .

We introduce the following notation which is necessary in the subsequent sections of the paper:

Definition 1: A function $f(\rho)$ is said to be exponentially equal to b, denoted by $f(\rho) \doteq \rho^b$, if

$$\lim_{\rho\to\infty} \frac{\log f(\rho)}{\log \rho} = b. \tag{23}$$

We can define the relation $\dot{\leq}$ in a similar fashion.

Definition 2: The exponential order of a random variable X with a non-negative support is given by,

$$V = -\lim_{\rho\to\infty} \frac{\log X}{\log \rho}. \tag{24}$$

The exponential order greatly simplifies the analysis of outage events while deriving the diversity multiplexing tradeoff. Some properties of the exponential order are derived in Appendix II, lemma 2.

Definition 3: (**Diversity-Multiplexing Tradeoff**) We use the definition given in [1], [12]. Consider a family of codes C_ρ operating at SNR ρ and having rates $R(\rho)$ bits per channel use. If $P_e(R)$ is the outage probability (see [10]) of the channel for rate R, then the multiplexing gain r and diversity order d are defined as[2]

$$r \triangleq \lim_{\rho\to\infty} \frac{R(\rho)}{\log \rho} \qquad\qquad d \triangleq -\lim_{\rho\to\infty} \frac{\log P_e(R)}{\log \rho} \tag{25}$$

B. Digital Relaying - Decode and Forward Protocol

We will first study the case where the intermediate relays have the ability to decode the received signal, re-encode and transmit it to the destination. We will study the protocols proposed in [8] and show that both these protocols can be considerably simplified through the selection relaying algorithm.

The decode and forward algorithm considered in [8] can be briefly summarized as follows. In the first half time-slots the source transmits and all the relays and receiver nodes listen to this transmission. Thereafter the relays that are successful in decoding the message, re-encode the message using a distributed space time protocol and collaboratively transmit it to the destination. The destination decodes the message at the end of the second time-slot. Note that the source does not transmit in the second half time-slots. The main result for the decode and forward protocol is given in the following theorem:

Theorem 2 ([8]): The achievable diversity multiplexing tradeoff for the decode and forward strategy with $N-1$ intermediate relay nodes is given by $d(r) = N(1-2r)$ for $r \in (0, 0.5)$.

The main advantage of opportunistic relaying in the decode and forward strategy is that it simplifies space-time coding since we can implement a practical space time code, like the Alamouti code across

[2]We will assume that the block length of the code is large enough, so that the detection error is arbitrarily small and the main error event is due to outage.

the source and the selected relay. Alamouti code has a useful property that any standard AWGN code can be used for space time transmission while preserving the optimality properties [11]. The following Theorem shows that opportunistic relaying achieves the same diversity-multiplexing tradeoff if the best relay selected according to rule 1.

Theorem 3: Under opportunistic relaying, the decode and forward protocol with $N-1$ intermediate relays achieves the same diversity multiplexing tradeoff stated in Theorem 2.

Proof: We follow along the lines of [8]. Let $\mathcal{E}$ denote the event that the relay is successful in decoding the message at the end of the first half of transmission and $\bar{\mathcal{E}}$ denote the event that the relay is not successful in decoding the message. Event $\bar{\mathcal{E}}$ happens when the mutual information between source and best relay drops below the code rate. Suppose that we select a code with rate $R = r\log\rho$ and let $I(X;Y)$ denote the mutual information between the source and the destination. The probability of outage is given by

$$\begin{aligned}
P_e &= \Pr(I(X;Y) \le r\log\rho|\mathcal{E})\Pr(\mathcal{E}) + \Pr(I(X;Y) \le r\log\rho|\bar{\mathcal{E}})\Pr(\bar{\mathcal{E}}) \\
&= \Pr\left(\frac{1}{2}\log(1+\rho(|a_{sd}|^2+|a_{rd}|^2)) \le r\log\rho\right)\Pr(\mathcal{E}) + \\
&\quad \Pr\left(\frac{1}{2}\log(1+\rho|a_{sd}|^2) \le r\log\rho\right)\Pr(\bar{\mathcal{E}}) \\
&\le \Pr\left(\frac{1}{2}\log(1+\rho(|a_{sd}|^2+|a_{rd}|^2)) \le r\log\rho\right) + \\
&\quad \Pr\left(\frac{1}{2}\log(1+\rho|a_{sd}|^2) \le r\log\rho\right)\Pr\left(\frac{1}{2}\log(1+\rho|a_{sr}|^2) \le r\log\rho\right) \\
&\le \Pr\left(|a_{sd}|^2+|a_{rd}|^2 \le \rho^{2r-1}\right) + \Pr\left(|a_{sd}|^2 \le \rho^{2r-1}\right)\Pr\left(|a_{sr}|^2 \le \rho^{2r-1}\right) \\
&\le \Pr\left(|a_{sd}|^2 \le \rho^{2r-1}\right)\Pr\left(|a_{rd}|^2 \le \rho^{2r-1}\right) + \Pr\left(|a_{sd}|^2 \le \rho^{2r-1}\right)\Pr\left(|a_{sr}|^2 \le \rho^{2r-1}\right) \\
&\dot{\le} \rho^{2r-1}\rho^{(N-1)(2r-1)} + \rho^{2r-1}\rho^{(N-1)(2r-1)} \doteq \rho^{N(2r-1)}
\end{aligned}$$

In the last step we have used claim 2 of Lemma 3 in the appendix with $m = N-1$[3].

■

C. Analog relaying - Basic Amplify and Forward

We will now consider the case where the intermediate relays are not able to decode the message, but can only scale their received transmission (due to the power constraint) and send it to the destination.

[3]in the previous section, we used notation M for the number of relays. Here we follow notation $N-1$ as in [8]. Therefore, $N-1 \equiv M$ throughout this work.

The basic amplify and forward protocol was studied in [7]. The source broadcasts the message for first half time-slots. In the second half time-slots the relay simply amplifies the signals it received in the first half time-slots. Thus the destination receives two copies of each symbol. One directly from the source and the other via the relay. At the end of the transmission, the destination then combines the two copies of each symbol through a matched filter. Assuming i.i.d Gaussian codebook, the mutual information between the source and the destination can be shown to be [7],

$$I(X;Y) = \frac{1}{2}\log\left(1+\rho|a_{sd}|^2 + f(\rho|a_{sr}|^2, \rho|a_{rd}|^2)\right) \tag{26}$$

$$f(a,b) = \frac{ab}{a+b+1} \tag{27}$$

The amplify and forward strategy does not generalize in the same manner as the decode and forward strategy does. We do not gain by having all the relay nodes amplify in the second half of the time-slot. This is because at the destination we do not receive a coherent summation of the channel gains from the different receivers. If β_j is the scaling constant of receiver j, then the received signal will be given by $y[n] = \left(\sum_{j=1}^{N-1} \beta_j a_{rd}^j\right) x[n] + z[n]$. Since this is simply a linear summation of Gaussian random variables, we do not see the diversity gain from the N relays. A possible alternative is to have the N relays amplify in a round-robin fashion. Each relay transmits only one out of every N symbols in a round robin fashion. This strategy has been proposed in [7], but the achievable diversity-multiplexing tradeoff is not analyzed. Opportunistic relaying provides another possible solution, where we only dedicate the best relay (according to rule 1) for transmission between the source to the destination. The following theorem shows that opportunistic relaying achieves the same diversity multiplexing tradeoff as that achieved by the more complicated decode and forward scheme in [7].

Theorem 4: Opportunistic amplify and forward achieves the same diversity multiplexing tradeoff stated in Theorem 2.

Proof: We begin with the expression for mutual information between the source and destination

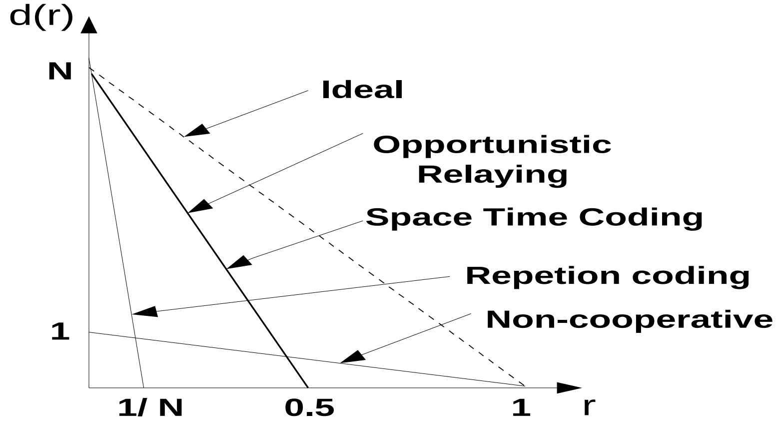

Fig. 6. The diversity-multiplexing of opportunistic relaying is exactly the same with that of more complex space time coded protocols.

(26). An outage occurs if this mutual information is less than the code rate $r \log \rho$. Thus we have that

$$\begin{aligned} P_e &= \Pr\left(I(X;Y) \le r\log\rho\right) \\ &= \Pr\left(\log(1+\rho|a_{sd}|^2 + f(\rho|a_{sr}|^2, \rho|a_{rd}|^2) \le 2r\log\rho\right) \\ &\le \Pr\left(|a_{sd}|^2 \le \rho^{2r-1}, f(\rho|a_{sr}|^2, \rho|a_{rd}|^2) \le \rho^{2r}\right) \\ &\overset{(a)}{\le} \Pr\left(|a_{sd}|^2 \le \rho^{2r-1}, \min\left(|a_{sr}|^2, |a_{rd}|^2\right) \le \rho^{2r-1} + \rho^{r-1}\sqrt{1+\rho^{2r}}\right) \\ &\overset{(b)}{\doteq} \rho^{2r-1}\rho^{(N-1)(2r-1)} = \rho^{N(2r-1)} \end{aligned}$$

Here (a) follows from Lemma 4 and (b) follows from Lemma 3, claim 1 in appendix II and the fact that $\rho^{r-1}\sqrt{1+\rho^{2r}} \rightarrow \rho^{2r-1}$ as $\rho \rightarrow \infty$. ■

D. Discussion

The diversity-multiplexing tradeoff calculated above is plotted in figure 6. Even though a single terminal with the ”best” end-to-end channel conditions (equations 1, 2) relays the information, the diversity order at the low SNR (ρ) regime is on the order of the number N of all participating terminals. Moreover, the tradeoff is exactly the same with that when space-time coding across $N-1$ relays is used.

From figure 6, it can be seen that performance is significantly better than *repetition coding* but deteriorates for multiplexing gain $r > 0.5$ where there is a significant gap from the ideal case (i.e. the trade-off achieved by a single terminal with N antennas transmitting to a single antenna receiver). That gap in performance occurs because we haven’t allowed the transmitter to transmit while the intermediate terminals transmit their relayed information. The basic incentive behind the above requirement is that we assume half-duplex relays, so that they could not receive while they transmit.

More complex protocols that allow the transmitter to continuously transmit were proposed in [1] and seem to close the aforementioned gap in the tradeoff performance. Opportunistic relaying could further simplify those protocols and details of such simplification and its performance are underway and will be reported elsewhere. The focus in this work is to show that opportunistic relaying simplifies cooperative diversity. Necessary coordination among cooperating nodes as well as practical distributed space-time coding (since maximum two nodes could in principle transmit simultaneously under opportunistic relaying[4]) are facilitated without performance loss, when compared with other approaches in the field [8].

VI. Conclusion

We proposed Opportunistic Relaying as a practical scheme for cooperative diversity. The scheme relies on distributed path selection considering end-to-end wireless channel conditions, facilitates coordination among the cooperating terminals and simplifies to practice necessary space-time coding in the communicating transceivers.

We presented a method to calculate the performance of the relay selection algorithm, for any king of wireless fading model and showed that successful relay selection could be engineered with reasonable performance. Specific examples for Rayleigh and Ricean fading were given.

Treating Opportunistic Relaying as a distributed virtual antenna array system and analyzed its diversity-multiplexing tradeoff revealed no performance loss when compared with more complex protocols in the field.

The approach presented in this work explicitly addresses coordination among the cooperating terminals and has similarities with a Medium Access Protocol (MAC) since it directs *when* a specific node to relay. The algorithm has also similarities with a Routing Protocol since it coordinates *which* node to relay (or not) received information among a collection of candidates. Devising wireless systems that dynamically adapt to the wireless channel conditions, in a distributed manner, similarly to the ideas presented in this work, is an important and fruitful area for future research.

The simplicity of the technique, allows for immediate implementation in existing radio hardware and its adoption could provide for improved flexibility, reliability and efficiency in future 4G wireless systems.

[4]the ”best relay” plus a transmitter.

APPENDIX I

PROBABILISTIC ANALYSIS OF SUCCESSFUL PATH SELECTION

Theorem 1 The joint probability density function of the minimum and second minimum among M i.i.d. positive random variables $X_1,\ X_2,\dots,\ X_M$, each with probability density function $f(x)$ and cumulative distribution function $F(x)$, is given by the following equation:

$$\begin{aligned} f_{Y_1,Y_2}(y_1,y_2) &= M\ (M-1)\ f(y_1)\ f(y_2)\ [1-F(y_2)]^{M-2} \\ &\quad 0<y_1<y_2, \\ f_{Y_1\ Y_2}(y_1,y_2) &= 0 \\ &\quad elsewhere. \end{aligned} \tag{28}$$

where $Y_1 < Y_2 < Y_3 \ldots < Y_M$ are the M ordered random variables $X_1,\ X_2,\dots,\ X_M$.

Proof: $f_{Y_1,Y_2}(y_1,y_2)\ dy1\ dy2 = Pr(Y_1\ \in\ dy_1,\ Y_2\ \in dy_2) =$

$$Pr(one\ X_i\ in\ dy_1,\ one\ X_j\ in\ dy_2\ (with\ y_2>y_1\ and\ i\neq j),\ and\ all\ the\ rest\ X_i's\ greater\ than\ y_2) =$$

$$= \ 2\ \binom{M}{2}\ Pr(\ X_1 \in dy_1,\ X_2 \in dy_2\ (y_2>y_1),\ X_i>y_2,\ i\in\ [3,M])\ =$$

$$= \ 2\ \binom{M}{2} f(y_1)\ dy_1\ f(y_2)\ dy_2\ [1-F(y_2)]^{M-2} =$$

$$= \ M\ (M-1)\ f(y_1)\ f(y_2)\ [1-F(y_2)]^{M-2}\ dy_1\ dy_2,\ for\ 0<y_1<y_2.$$

The third equality is true since there are $\binom{M}{2}$ pairs in a set of M i.i.d. random variables. The factor 2 comes from the fact that ordering in each pair matters, hence we have a total number of $2\ \binom{M}{2}$ cases, with the same probability, assuming identically distributed random variables. That concludes the proof. ∎

Using Theorem 1, we can prove the following lemma:

Lemma 1 Given M i.i.d. positive random variables $X_1,\ X_2,\dots,\ X_M$, each with probability density function $f(x)$ and cumulative distribution function $F(x)$, and $Y_1 < Y_2 < Y_3 \ldots < Y_M$ the M ordered random variables $X_1,\ X_2,\dots,\ X_M$, then $Pr(\ Y_2 < Y_1 + c)$, where $c > 0$, is given by the following

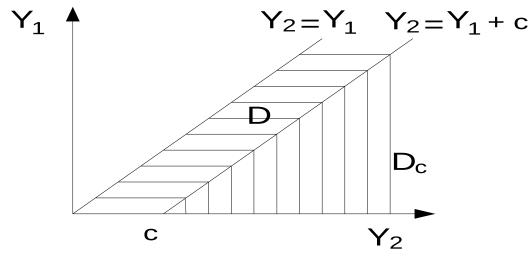

Fig. 7. Regions of integration of $f_{Y_1,Y_2}(y_1, y_2)$, for $Y_1 < Y_2$ needed in Lemma I for calculation of $Pr(Y_2 < Y_1 + c),\ c > 0$.

equations:

$$Pr(Y_2 < Y_1 + c) = 1 - I_c \tag{29}$$

$$I_c = M\ (M-1)\ \int_c^{+\infty} f(y)\ [1 - F(y)]^{M-2}\ F(y-c)\ dy \tag{30}$$

Proof: The joint pdf $f_{Y_1,Y_2}(y_1, y_2)$ integrates to 1 in the region $D \cup D_c$, as it can be seen in figure 7. Therefore:

$$\begin{aligned} Pr(\ Y_2 < Y_1 + c) &= \int\int_D f_{Y_1,Y_2}(y_1, y_2)\ dy1\ dy2 \\ &= 1 - \int\int_{D_c} f_{Y_1,Y_2}(y_1, y_2)\ dy1\ dy2 \\ &= 1 - I_c \end{aligned}$$

Again from figure 7, I_c can easily be calculated:

$$\begin{aligned} I_c = \\ {\scriptstyle M(M-1)} \int_{y_2=c}^{+\infty} f(y_2)\ [1 - F(y_2)]^{M-2}\ \int_0^{y_2-c} f(y_1)\ dy_1\ dy_2 \\ = {\scriptstyle M(M-1)} \int_{y_2=c}^{+\infty} f(y_2)\ [1 - F(y_2)]^{M-2}\ F(y_2 - c)\ dy_2 \end{aligned} \tag{33}$$

The last equation concludes the proof. ■

Appendix II

Diversity-Multiplexing Tradeoff Analysis

We repeat *Definition 1* and *Definition 2* in this section for completeness. The relevant lemmas follow.

Definition 1: A function $f(\rho)$ is said to be exponentially equal to b, denoted by $f(\rho) \doteq \rho^b$, if

$$\lim_{\rho \to \infty} \frac{\log f(\rho)}{\log \rho} = b. \tag{34}$$

We can define the relation $\dot{\leq}$ in a similar fashion.

Definition 2: The exponential order of a random variable X with a non-negative support is given by,

$$V = -\lim_{\rho\to\infty} \frac{\log X}{\log \rho}. \tag{35}$$

Lemma 2: Suppose $X_1, X_2, \ldots, X_m$ are m i.i.d exponential random variables with parameter λ (mean $1/\lambda$), and $X = \max\{X_1, X_2, \ldots X_m\}$. If V is the exponential order of X then the density function of V is given by

$$f_V(v) \doteq \begin{cases} \rho^{-mv} & v \geq 0 \\ 0 & v < 0 \end{cases} \tag{36}$$

and

$$\Pr(X \leq \rho^{-v}) \doteq \rho^{-mv} \tag{37}$$

Proof: Define,

$$V_\rho = -\frac{\log X}{\log \rho}.$$

Thus V_ρ is obtained from definition 2, without the limit of $\rho \to \infty$.

$$\begin{aligned} \Pr(V_\rho \geq v) &= \Pr(X \leq \rho^{-v}) \\ &= \Pr(X_1 \leq \rho^{-v}, X_2 \leq \rho^{-v}, \ldots X_m \leq \rho^{-v}) \\ &= \prod_{i=1}^{m} \Pr(X_i \leq \rho^{-v}) \\ &= \big(1 - \exp(-\lambda\rho^{-v})\big)^m \\ &= \left(\lambda\rho^{-v} + \sum_{j=2}^{\infty} \frac{(-\lambda)^j}{j!}\ \rho^{-jv}\right)^m \end{aligned}$$

Note that $\Pr(V_\rho \geq v) \approx \rho^{-mv}$. Differentiating with respect to v and then taking the limit $\rho \to \infty$, we recover (36). ∎

From the above it can be seen that for the simple case of a single exponential random variable ($m = 1$), $\Pr(X \leq \rho^{-v}) = \Pr(V_\rho \geq v) \doteq \rho^{-v}$.

Lemma 3: For relays, $j = 1, 2, \ldots, m$, let a_{sj} and a_{jd} denote the channel gains from source to relay j and relay j to destination. Suppose that a_{sr} and a_{rd} denote the channel gain of the source to the best relay and the best relay to the destination, where the relay is chosen according to rule 1. i.e.

$$\min(|a_{sr}|^2, |a_{rd}|^2) = \max\{\min(|a_{s1}|^2, |a_{1d}|^2), \ldots, \min(|a_{sm}|^2, |a_{md}|^2)\}$$

Then,

1) $\min(|a_{sr}|^2, |a_{rd}|^2)$ has an exponential order given by (36).

2)

$$\Pr(|a_{sr}|^2 \leq \rho^{-v}) = \Pr(|a_{rd}|^2 \leq \rho^{-v}) \dot{\leq} \begin{cases} \rho^{-mv} & v \geq 0 \\ 1 & \text{otherwise} \end{cases}$$

Proof: Let us denote $X^{(j)} \triangleq \min(|a_{sj}|^2, |a_{jd}|^2)$. Since each of the $X^{(j)}$ are exponential random variables with parameter 2, claim 1 follows from Lemma 2. Also since $|a_{sd}|^2$ and $|a_{rd}|^2$ cannot be less than $\min(|a_{sd}|^2, |a_{rd}|^2)$ claim 2 follows immediately from claim 1. ■

Lemma 4: With $f(\cdot, \cdot)$ defined by relation (27), we have that

$$\Pr\left(f(\rho a, \rho b) \leq \rho^{2r}\right) \leq \Pr\left(\min(a,b) \leq \rho^{2r-1} + \rho^{r-1}\sqrt{1+\rho^{2r}}\right).$$

Proof: Without loss in generality, assume that $a \geq b$.

$$\begin{aligned} f(\rho a, \rho b) &= \rho \frac{ab}{a+b+\frac{1}{\rho}} \\ &= \rho b \left(\frac{a}{a+b+\frac{1}{\rho}}\right) \\ &\overset{(a)}{\geq} \rho b \left(\frac{b}{2b+\frac{1}{\rho}}\right) \end{aligned}$$

Here (a) follows since $\frac{a}{a+K}$ is an increasing function in a, for $K > 0$ and $a \geq b$.

Now we have that

$$\begin{aligned} \Pr\left(f(\rho a, \rho b) \leq \rho^{2r}\right) &\leq \Pr\left(\frac{b^2}{2b+\frac{1}{\rho}} \leq \rho^{2r-1}\right) \\ &= \Pr\left(b^2 \leq 2\rho^{2r-1} b + \rho^{2r-2}\right) \\ &= \Pr\left((b-\rho^{2r-1})^2 \leq \rho^{4r-2} + \rho^{2r-2}\right) \\ &\overset{(a)}{=} \Pr\left(b \leq \rho^{2r-1} + \rho^{r-1}\sqrt{1+\rho^{2r}}\right) \end{aligned}$$

Where (a) follows since $b \geq 0$ so that $\Pr(b < 0) = 0$.

■

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