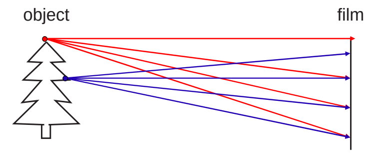
Lecture 9

Image Formation

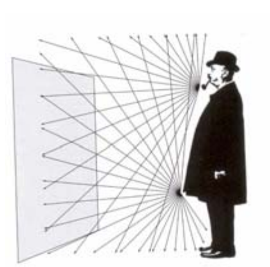
How do we see the world?



- Let's design a camera
  - Idea 1: put a piece of film in front of an object
  - Do we get a reasonable image?

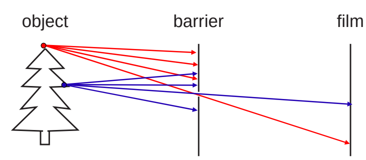
Slide by Steve Seitz

It receives light from all directions



From Photography, London et al.

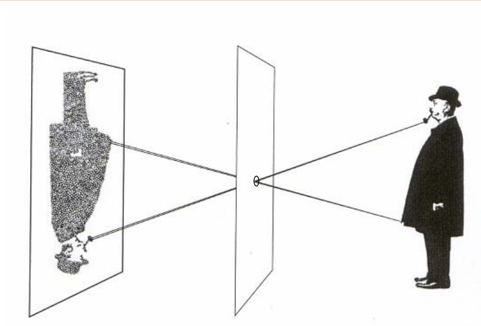
Pinhole camera



- Add a barrier to block off most of the rays
  - This reduces blurring
  - The opening known as the **aperture**
  - How does this transform the image?

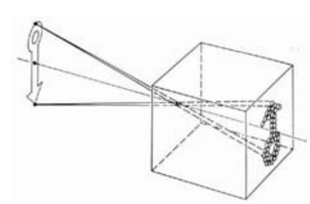
Slide by Steve Seitz

Pinhole



From Photography, London et al.

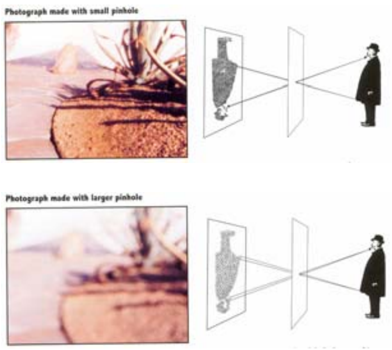
Pinhole camera model



- Pinhole model:
  - Captures **pencil of rays** – all rays through a single point
  - The point is called **Center of Projection (COP)**
  - The image is formed on the **Image Plane**
  - **Effective focal length *f*** is distance from COP to Image Plane

Slide by Steve Seitz

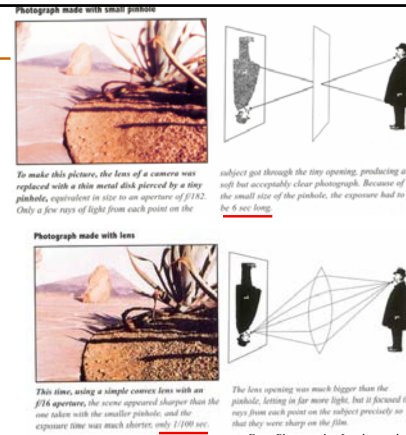
Pinhole size?



From Photography, London et al.

Lenses

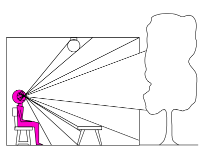
- gather more light!
- But need to be focused

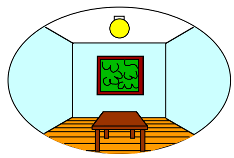


From Photography, London et al.

Dimensionality Reduction Machine (3D to 2D)

3D world *2D image*





Point of observation

- What have we lost?
  - Angles
  - Distances (lengths)

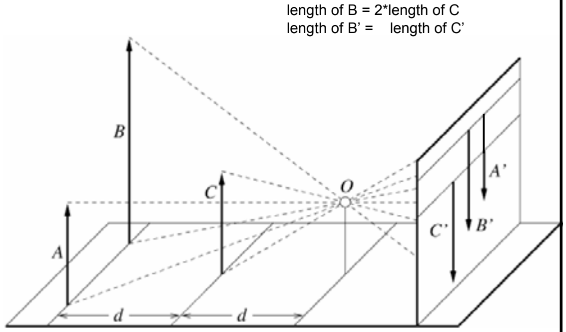
Figures © Stephen E. Palmer, 2002

Funny things happen…

Parallel lines aren't…

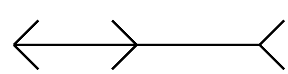
Distant objects are smaller

length of B = 2*length of C
length of B' = length of C'

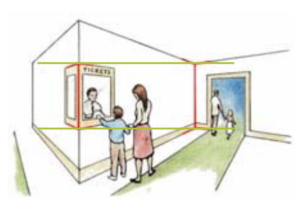


Forsyth&Ponce

…but humans adopt!



Müller-Lyer Illusion

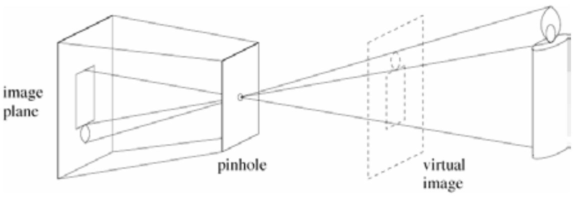


We don't make measurements in the image plane

http://www.michaelbach.de/ot/sze_muelue/index.html

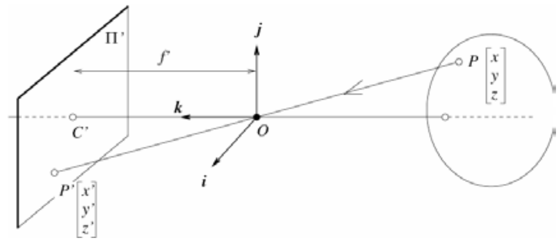
Perspective projection

- **Abstract camera model - box with a small hole in it**
- **In an ideal pinhole camera everything is in focus**



Forsyth&Ponce

The equation of projection



2D image *3D world*

The equation of projection

- Cartesian coordinates:
  - We have, by similar triangles, that

$$(x,y,z) \rightarrow (f\frac{x}{z}, f\frac{y}{z}, -f)$$

  - Ignore the third coordinate, and get

$$(x,y,z) \rightarrow (f\frac{x}{z}, f\frac{y}{z})$$

The camera matrix

- Turn previous expression into HC's
  - HC's for 3D point are *(x,y,z,1)*
  - HC's for point in image are *(u,v,w)*

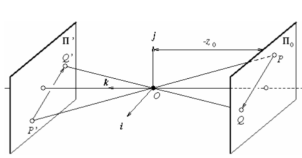
$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

- Position of the point in the image from HC

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} x \\ y \\ z/f \end{pmatrix} \xrightarrow{\text{normalize by } w} \frac{1}{w}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \frac{f}{z}\begin{pmatrix} x \\ y \\ z/f \end{pmatrix} = \begin{pmatrix} fx/z \\ fy/z \\ 1 \end{pmatrix}$$

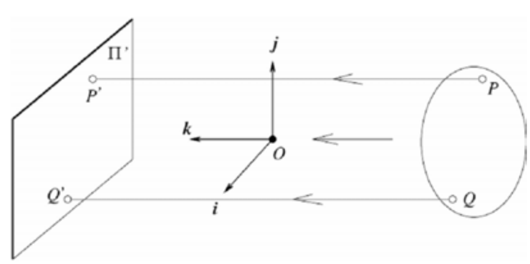
Weak perspective

- Issues:
  - perspective effects, but not over the scale of individual objects
  - collect points into a group at about the same depth, then divide each point by the depth of its group
  - Adv: easy
  - Disadv: wrong



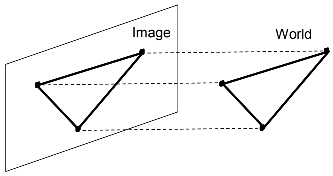
Orthographic projection

Telescope projection can be modeled by orthographic projection



Orthographic Projection

- Special case of perspective projection
  - Distance from the COP to the PP is infinite



  - Also called “parallel projection”
  - What’s the projection matrix?

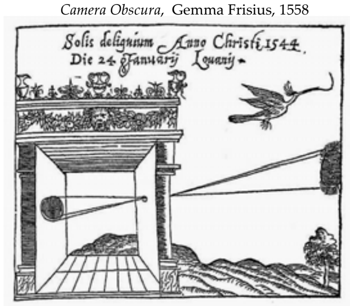
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Rightarrow (x, y)$$

Slide by Steve Seitz

Building a real camera

Camera Obscura

Camera Obscura, Gemma Frisius, 1558



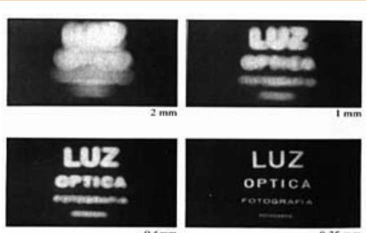
- The first camera
  - Known to Aristotle
  - Depth of the room is the effective focal length

Home-made pinhole camera

Why so blurry?

http://www.debevec.org/Pinhole/

Shrinking the aperture



- Why not make the aperture as small as possible?
  - Less light gets through
  - Diffraction effects…

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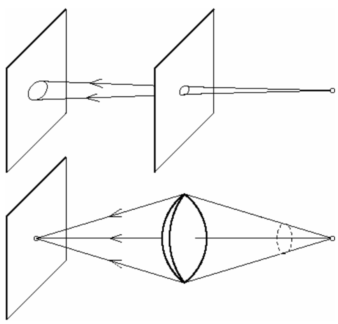
Shrinking the aperture

The reason for lenses



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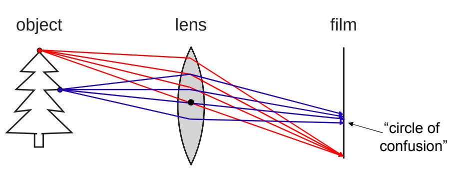
Focus

Focus and Defocus



- A lens focuses light onto the film
  - There is a specific distance at which objects are "in focus"
    - other points project to a "circle of confusion" in the image
  - Changing the shape of the lens changes this distance

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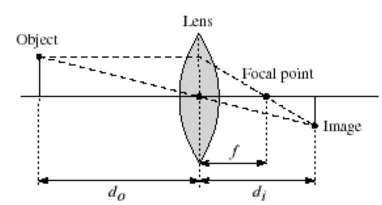
Thin lenses



- Thin lens equation: $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$
  - Any object point satisfying this equation is in focus
  - What is the shape of the focus region?
  - How can we change the focus region?
  - Thin lens applet: http://www.phy.ntnu.edu.tw/java/Lens/lens_e.html (by Fu-Kwun Hwang)

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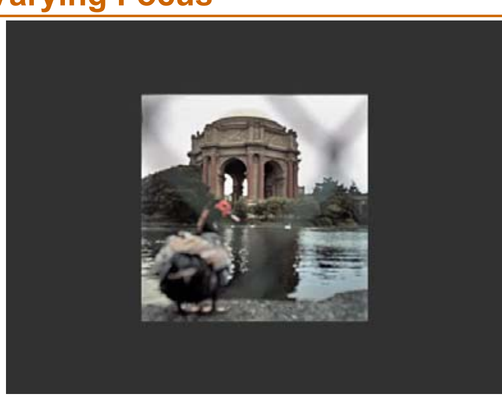
Varying Focus



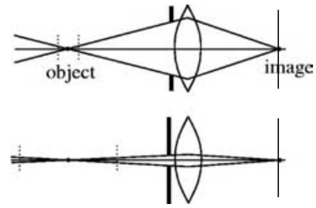
Ren Ng

Depth Of Field

Depth of Field

http://www.cambridgeincolour.com/tutorials/depth-of-field.htm

Aperture controls Depth of Field



- Changing the aperture size affects depth of field
  - A smaller aperture increases the range in which the object is approximately in focus
  - But small aperture reduces amount of light – need to increase exposure

Varying the aperture

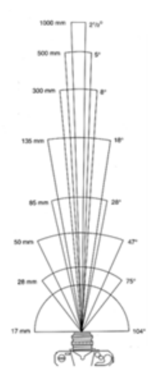
Large apeture = small DOF

Small apeture = large DOF

Nice Depth of Field effect

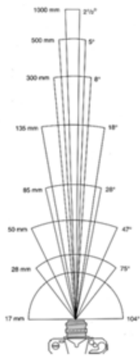
Field of View (Zoom)

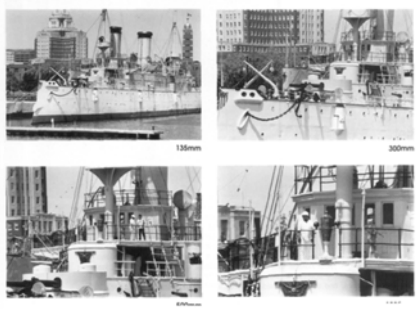
Field of View (Zoom)



From London and Upton

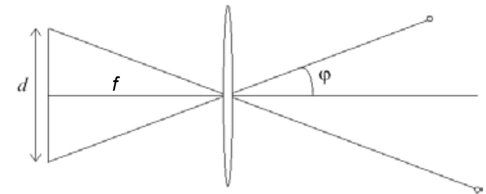
Field of View (Zoom)





From London and Upton

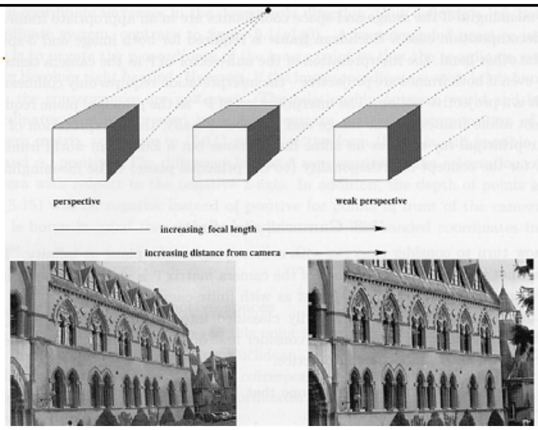
FOV depends of Focal Length



Size of field of view governed by size of the camera retina:

$$\varphi = \tan^{-1}\left(\frac{d}{2f}\right)$$

Smaller FOV = larger Focal Length



From Zisserman & Hartley

Field of View / Focal Length

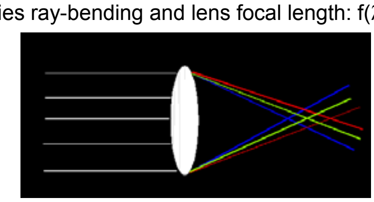
Large FOV, small f
Camera close to car

Small FOV, large f
Camera far from the car

Lens Flaws

Lens Flaws: Chromatic Aberration

- Dispersion: wavelength-dependent refractive index
  - (enables prism to spread white light beam into rainbow)
- Modifies ray-bending and lens focal length: f(λ)



- color fringes near edges of image
- Corrections: add ‘doublet’ lens of flint glass, etc.

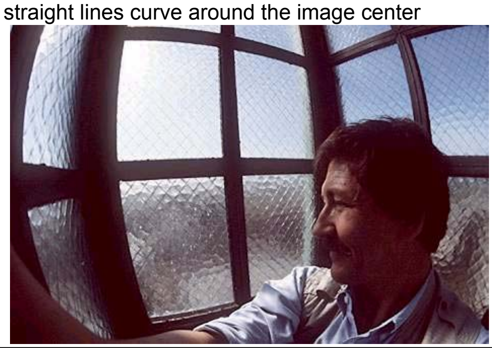
Chromatic Aberration

Near Lens Center

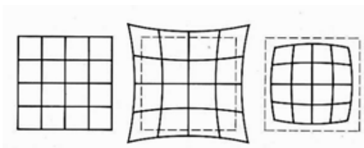
Near Lens Outer Edge

Radial Distortion (*e.g.* 'Barrel' and 'pin-cushion')

straight lines curve around the image center



Radial Distortion



- Radial distortion of the image
  - Caused by imperfect lenses
  - Deviations are most noticeable for rays that pass through the edge of the lens

Radial Distortion