Lecture 9
Image Formation

How do we see the world?

Let's design a camera
- Idea 1: put a piece of film in front of an object
- Do we get a reasonable image?

It receives light from all directions

Add a barrier to block off most of the rays
- This reduces blurring
- The opening known as the aperture
- How does this transform the image?

Pinhole camera model

- Pinhole model:
  - Captures pencil of rays – all rays through a single point
  - The point is called Center of Projection (COP)
  - The image is formed on the Image Plane
  - Effective focal length $f$ is distance from COP to Image Plane
Pinhole size?

From Photography, London et al.

Lenses

- gather more light!
- But need to be focused

From Photography, London et al.

Dimensionality Reduction Machine
(3D to 2D)

3D world

2D image

- What have we lost?
  - Angles
  - Distances (lengths)

From Photography, London et al.

Funny things happen…

Parallel lines aren’t…

Distant objects are smaller

…but humans adopt!

Müller-Lyer Illusion

We don’t make measurements in the image plane

http://www.michaelbach.de/ot/sze_muelue/index.html
Perspective projection

- Abstract camera model - box with a small hole in it
- In an ideal pinhole camera everything is in focus

\[
\begin{align*}
\text{image plane} & \quad \text{pinhole} & \quad \text{virtual image}
\end{align*}
\]

The equation of projection

- Cartesian coordinates:
  - We have, by similar triangles, that
    \[
    (x, y, z) \rightarrow \left( f \frac{x}{z}, f \frac{y}{z}, -f \right)
    \]
  - Ignore the third coordinate, and get
    \[
    (x, y, z) \rightarrow \left( f \frac{x}{z}, f \frac{y}{z} \right)
    \]

The camera matrix

- Turn previous expression into HC’s
  - HC’s for 3D point are \((x, y, z, 1)\)
  - HC’s for point in image are \((u, v, w)\)

\[
\begin{pmatrix}
\frac{x}{f} \\
\frac{y}{f} \\
\frac{z}{f}
\end{pmatrix}
\]

- Position of the point in the image from HC

\[
\begin{pmatrix}
u \\
v \\
w
\end{pmatrix}
= \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & f/f_w & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
x \\
y \\
z \\
1
\end{pmatrix}
\]

Weak perspective

- Issues:
  - perspective effects, but not over the scale of individual objects
  - collect points into a group at about the same depth, then divide each point by the depth of its group
  - Adv: easy
  - Disadv: wrong

Orthographic projection

Telescope projection can be modeled by orthographic projection
Orthographic Projection

- Special case of perspective projection
  - Distance from the COP to the PP is infinite

  Also called “parallel projection”
  What’s the projection matrix?

\[
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
x' \\
y' \\
z' \\
1
\end{pmatrix}
\Rightarrow
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix}
\]

Building a real camera

Camera Obscura

- The first camera
  - Known to Aristotle
  - Depth of the room is the effective focal length

Camera Obscura, Gemma Frisius, 1558

Shrinking the aperture

- Why not make the aperture as small as possible?
  - Less light gets through
  - Diffraction effects…

Home-made pinhole camera

Why so blurry?

http://www.debevec.org/Pinhole/
The reason for lenses

A lens focuses light onto the film
- There is a specific distance at which objects are “in focus”
- Other points project to a “circle of confusion” in the image
- Changing the shape of the lens changes this distance

Focus and Defocus

Focus and Defocus

Thin lenses

- Thin lens equation: \( \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \)
  - Any object point satisfying this equation is in focus
  - What is the shape of the focus region?
  - How can we change the focus region?
  - Thin lens applet: [http://www.phy.ntnu.edu.tw/java/Lens/lens_e.html](http://www.phy.ntnu.edu.tw/java/Lens/lens_e.html)

Varying Focus

Depth Of Field
Depth of Field

http://www.cambridgeincolour.com/tutorials/depth-of-field.htm

Changing the aperture size affects depth of field
- A smaller aperture increases the range in which the object is approximately in focus
- But small aperture reduces amount of light – need to increase exposure

Varying the aperture

Large aperture = small DOF  Small aperture = large DOF

Nice Depth of Field effect

Field of View (Zoom)

From London and Upton
**Field of View (Zoom)**

*From London and Upton*

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**FOV depends of Focal Length**

Size of field of view governed by size of the camera retina:

\[ \varphi = \tan^{-1} \left( \frac{d}{2f} \right) \]

Smaller FOV = larger Focal Length

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**Field of View / Focal Length**

*From Zisserman & Hartley*

Large FOV, small \( f \) - Camera close to car  
Small FOV, large \( f \) - Camera far from the car

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**Lens Flaws: Chromatic Aberration**

- Dispersion: wavelength-dependent refractive index  
  - (enables prism to spread white light beam into rainbow)  
  - Modifies ray-bending and lens focal length: \( f(\lambda) \)

- color fringes near edges of image  
- Corrections: add ‘doublet’ lens of flint glass, etc.
**Chromatic Aberration**

- Near Lens Center
- Near Lens Outer Edge

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**Radial Distortion**

- Radial distortion of the image
  - Caused by imperfect lenses
  - Deviations are most noticeable for rays that pass through the edge of the lens

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**Radial Distortion**

- Radial distortion: straight lines curve around the image center

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**Radial Distortion**

- Examples: 'Barrel' and 'pin-cushion'