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SYNTHESIS OF PARALLEL MANIPULATORS USING LIE-GROUPS
Y-STAR AND H-ROBOT

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ABSTRACT

Our aim is to give a complete presentation of the application of Lie Group Theory to the structural design of manipulator robots. We focused our attention on parallel manipulator robots and in particular those capable of spatial translation. This is justified by many industrial applications which do not need the orientation of the end-effector in the space. The advantage of this method is that we can derive systematically all kinematics chains which produce the desired displacement subgroup. In addition, we obtain kinematic equations with ease and rapidity. Hence, an entire family of robots results from our investigation. The Y-STAR manipulator was constructed at the Ecole Centrale Paris and is now a working device. Learning techniques are currently being studied to enhance his capabilities. H-ROBOT is also being constructed. Both manipulators respond to the increasing demand of fast working rhythms in modern production at a low cost and are suited for any kind of pick and place jobs like sorting, arranging on palettes, packaging and assembly.

1. INTRODUCTION

The mathematical theory of groups can be applied to the set of displacements. If we call \{ D \} the set of all possible displacements, it is proved, according to this theory, that \{ D \} had a group structure. The most remarkable movements of a rigid body are then represented by subgroups of \{ D \}. This method leads to a classification of mechanisms [1]. The main step for establishing such a classification is the derivation of an exhaustive inventory of the subgroups of the displacement group. This can be done by a direct reasoning by examining all the kinds of products of rotations and translations [2]. However, a much more effective method consists in using Lie Group Theory [3],[4]. Lie Groups are defined by analytical transformations depending on a finite number of real parameters. The displacement group \{ D \} is a special case of a Lie group of dimension six.

Our interest in Lie group theory is also confirmed by a general gain of interest in this field of mathematics by other researchers. These people utilize Lie groups in various different fields of application all related to robotics as for example nonlinear control theory [5] and vision [6], [7].

2. LIE'S THEORY

Within the framework of Lie's theory, we associate infinitesimal transformations making up a Lie algebra with finite operations which are obtained from the previous ones by exponentiation. Continuous analytical groups are described by the exponential of differential operators which correspond to the infinitesimal transformations of the group. Furthermore, group properties are interpreted by the algebraic structure of Lie algebra of the differential operators and conversely. We recall the main definition axiom of a Lie algebra: a Lie algebra is a vector space endowed with a bilinear skew symmetric inner product. It is well known [8], that the set of screw velocity fields is a vector space of dimension six for the natural operations at a given point N.

By following the steps indicated in [3] we can produce the exhaustive list of the Lie subgroups of the group of euclidean displacements \{ D \} (see synoptical list 1). This is done by first defining a differential operator associated with the velocity field. Then, by exponentiation, we derive the formal Lie expression of finite displacements which are shown to be equivalent to affine direct orthonormal transformations. Lie sub-algebras of screw velocity fields lead to the description of the displacement subgroups.

3. THE \( \{ X(w) \} \) SUBGROUP

In order to generate spatial translation with parallel mechanisms, we are led to look for displacement subgroups the intersection of which is the spatial translation subgroup \{ T \}. We will consider only the cases for which the intersection subgroup is strictly included in the two "parallel" subgroups. The most important case of this sort is the parallel association of two \( \{ X(w) \} \) subgroups with two distinct vector directions \( w \) and \( w' \). It is easy to prove:

\( \{ X(w) \} \cap \{ X(w') \} = \{ T \} \quad w \neq w' \)
The subgroup \{X(w)\} plays a prominent role in mechanism design. This subgroup combines spatial translation with rotation about a movable axis which remains parallel to a given direction \(w\). Physical implementations of \{X(w)\} mechanical liaisons can be obtained by putting in series kinematic pairs represented by subgroups of \{X(w)\}. Practically only prismatic, revolute and screw pairs P, R, H are used to build robots (the cylindric pair C combines in a compact way a prismatic pair and a revolute pair). A complete list of all possible combinations of these kinematic pairs generating the \{X(w)\} subgroup is given in [9].

Two geometrical conditions have to be satisfied in the series: the rotation axes and the screw axes are parallel to the given vector \(w\); there is no passive mobility.

The displacement operator for the \{X(w)\} subgroup, acting on point \(M\) is:

\[M \rightarrow N + au + bv + cw + \exp(hw) \wedge NM\]

Point \(N\) and the vectors \(u, v, w\) make up an orthogonal frame of reference in the space and \(a, b, c, h\) are the four parameters of the subgroup which has dimension 4.

The advantage of using this method to find mathematical expressions is that it is independent of the choice of a particular frame of reference.

4. PARALLEL ROBOTS FOR SPATIAL TRANSLATION

To produce spatial translation it is sufficient to place two mechanical generators of the subgroups \{X(w)\} and \{X(w')\}, \(w \neq w'\), in parallel, between a mobile platform and a fixed platform. If we want to build a robot which has only fixed motors then three generators of the three subgroups \{X(w)\}, \{X(w')\}, \{X(w'')\}, \(w \neq w', w' \neq w'', w'' \neq w\), are needed. Any series of P, R or H pairs which constitute a mechanical generator of the \{X(w)\} subgroup can be implemented. Moreover, these three mechanical generators may be different or the same depending on the desired kinematic results. This wide range of combinations gives rise to an entire family of robots capable of spatial translation. Simulation of the most interesting architectures can easily be achieved and the choice of the robot to be constructed can therefore meet the needs of the commissioner.

Clavel’s Delta robot belongs to this family as it is based on the same kinematic principles [10].

5. THE PARALLEL MANIPULATOR Y-STAR

STAR is made up by three cooperating arms which generate the subgroups \{X(u)\}, \{X(u')\}, \{X(u'')\} (fig 1). The three arms are identical and each one generates a subgroup \{X(u)\} by the series RHPaR where Pa represents the circular translation liaison determined by the two opposite bars of a plane hinged parallelogram. The axes of the two revolute pairs and of the screw pair must be parallel in order to generate a \{X(u)\} subgroup. Hence we write R1uHuPaR2u. For each arm, the first two pairs, i.e. the coaxial revolute pair and the screw pair, constitute the fixed part of the robot and form at the same time the mechanical structure of an electric jack which can be fixed in the frame. All the Hu axes lie on the same plane \(\Pi\) and divide it into three identical parts thus forming a Y shape. Hence the angle between any two axes is always 2\(\pi/3\). The mobile part of the robot is made up by the PaR2u series of each arm that all converge to a common point below which the mobile platform is located. The platform stays parallel to the reference plane \(\Pi\) and cannot rotate about the axis perpendicular to this plane. Any kind of appropriate end effector can be placed on this mobile platform.

![fig.1](image)

The three deformable parallelograms ensure the stability and increase the rigidity of the whole. As we have already mentioned above, the axis of the last revolute pair R2u has to be parallel to the one of the first revolute pair R1u. This theoretical condition needs to be carefully applied in practice.

The derivation of the \{T\} subgroup, which proves the the mobile platform can only translate in the space, is given in [11].

6. INVERSE KINEMATICS

To express the kinematic transformations of any given point \(M\) of the mobile platform bearing the end effector, we use the direct intrinsic vector method. We shall consider only two arms, as the two subgroups \{X(u)\} and \{X(u')\} are sufficient to produce the intersection subgroup \{T\}. As a first step, we cut the platform in two parts allowing independent operations of the two parallel arms. For the first arm we observe that each component pair permits only one degree of freedom represented by an angle that we shall put into brackets:
For the second arm we will therefore have: \( R_{1u}(\psi)H_u(\chi)P_{a}(\delta)R_{2u}(\psi) \). The first step of the direct intrinsic vector method is to describe the initial configuration. Easy computation can be obtained by choosing a convenient configuration. In this case, we have the arms in a vertical initial configuration. \((u,v,w)\) make up a direct orthogonal vector base, \( C \) is the center of the mobile platform, \( M \) is \( \omega x \) point belonging to the same platform and \( O \) is the point common to the three axes.

\[ CB = cu \quad and \quad AB = bw. \]

For technical reasons, in the practical implementation of the robot we should have \( c \) as small as possible whereas \( b \), the arm's length, is a parameter which can be optimized depending from the desired application and the consequent working volume.

In this second step, we will establish the symbolic model of our mechanism using the previous system of notation. For the first arm we have point \( M \) becoming \( M' \) according to the formula:

\[
\begin{bmatrix}
\exp(\psi u \Lambda)
\end{bmatrix}^{T}
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\exp(\chi u \Lambda)
\end{bmatrix}^{T}
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\exp(\psi u \Lambda)
\end{bmatrix}^{T}
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
BM
\end{bmatrix}
\]

After some calculations we obtain the inverse kinematic formula:

\[
di = x \cos \delta_i + y \sin \delta_i + \\
\sqrt{b^2 - x^2 - y^2 - z^2 + (\cos \delta_i + y \sin \delta_i)^2} - \arccos \frac{z}{\sqrt{x^2 + y^2 + z^2 - (\cos \delta_i + y \sin \delta_i)^2}}
\]

\[i = 1,2,3.\]

8. SINGULAR CONFIGURATIONS

The research of singular configurations is quite important. In fact, in this case the mobile platform would be displaced without any movement of the actuators [12] and the whole mechanism would not be under control any more. In addition, in the vicinity of a singular configuration, articualar forces may be considerable.

After many calculations, done also with the help of some software, only the two obvious singular configurations have been found:

1. \( z=0, x=b, y=b \);  
2. \( z=b, x=0, y=0 \).

The determinant of the inverse jacobian matrix is zero only in some other points situated on the border of the working volume but not inside it.

9. THE WORKING VOLUME

Y-STAR robot's working volume is represented in fig. 2:

![fig. 2](image_url)

The physical implementation of the robot we built has the following dimensions:

\( R_{a1} = b=500 \text{ mm}; \quad R_{a2} = 44 \text{ mm}; \quad L_{1} = 600 \text{ mm}. \)

One can think that the longer the arms, the greater the working volume. However if we take an axis' length of 600 mm, this is true up to 400 mm. For example for a 750 mm arm's length the working volume is very little. The parameter to be considered is then the ratio between the arm's length and the fixed axes' length. Always working with an axis of 600 mm long we observed that from a 300 mm arm's length up, the horizontal sections of the base of the working volume take the form of a star with 3 branches which get thinner and thinner as the length of the arm increases (fig. 3, fig. 4).

We concluded that the ideal ratio ranges from 1/2 to 2/3. This means in our case \( b = 350 \) to 400 mm. Our choice of having an arm 500 mm long may be justified for example
by security reasons: we obtain a relatively thin working area at the bottom and we mechanically restrict the access of the robot to certain positions in the space.

10. Y-STAR: MECHANICAL STRUCTURE

A small prototype was built at the Ecole Centrale Paris (fig. 5; fig. 6) in alluminium alloy (durall) and has the following parameters:

- $b = 500$ mm = arm's length;
- $l = 600$ mm = fixed axis' length;
- $k = 50.8$ mm = screw's pitch.

11. Y-STAR: CONTROL

The control method implemented for this first prototype is a classical proportional-integral-derivative (PID) method. A simple demonstration program was implemented to show Y-STAR at work. It makes the end-effector (a pencil) draw a precise trajectory on plan. The resulting image is the logotype of Ecole Centrale (fig. 7).

Six infrared sensors are placed at the extremities of each axes to ensure security. If the nut cuts the infrared beam, the motor amplifiers are desactivated. As the motors don't apply any more torque and the inertia being very small, this stops the robot very rapidly.

12. FUTURE IMPROVEMENTS

Technical improvements can be implemented: for example a second screw pair having the same pitch as the first may replace the second revolute pair R2. No correction of the positionning angle would then be necessary.

The choice of composite materials for the arms and the mobile platform will of course improve the efficiency of Y-STAR.
Further improvements concerning the global mechanical structure can be achieved by studying the working volume and the inverse kinematic problem of the other robots belonging to the same family as Y-STAR. Appropriate software was developed to simulate the different options obtained by changing the position of the fixed rotational axes in the plane. With the use of this software (written in C++) we examined H-ROBOT, Δ-ROBOT, U-ROBOT and T-ROBOT. The most interesting mechanical structure, alternative to Y-STAR resulted to be H-ROBOT. The construction of this robot has already started and some characteristics of this robot are presented in the following paragraph.

13. THE H-ROBOT

H-Robot is obtained from Y-STAR by changing the orientation of the fixed rotation axes. H-Robot is made up by 3 systems screws (1) / nut (2) with a large pitch, which allow rapid movements. It is hold by poles (6) and animated by the the actuators (M). Three plane hinged parallelograms, on both sides (4) and at the center (5) make the connection from the nuts to the horizontal platform (3). The stand (7) supports the whole structure (fig. 9).

The side screws allow rotation about their translation axis. The central nut does not permit the rotation of the plane hinged parallelogram about the screw axis. Each movement determines the deformation of the parallelograms. Hence the mobile platform can only translate in the working space. The working volume can be assimilated to a half-cylinder.

The main advantage of this device is that it has the three fixed rotation axis in parallel. Hence the working volume is directly proportional to the length of these axis and it can be made considerably large. We chose to have fixed rotation axis of 1 meter long.

All the actuators are in a fixed position. The three arm are stiff and lightweighted. The resulting movements of the mobile platform are then very fast.

H-Robot seems to be a better solution than Y-STAR especially in those cases where the size of the working volume becomes important.

CONCLUSIONS

The importance of Lie group theory, especially for kinematics is recognized from various sources [13], [14], [15], [16], [17], [18]. Investigation of new parallel robots generating pure translation led us to the construction of two prototypes: Y-STAR and H-ROBOT. Increasing performances and the low cost of fabrication make these robots attractive for modern industry. They are presented as an alternative to the DELTA robot and have the classical parallel robot advantages for positioning, precision, rapidity and fixed motor location.

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REFERENCES

## Sub-Groups of the Displacement Group

<table>
<thead>
<tr>
<th>NOTATION</th>
<th>TRANSFORMATION</th>
<th>VARIABLES OF Translation. Rotation</th>
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<tbody>
<tr>
<td>0</td>
<td>E</td>
<td>M→M</td>
</tr>
<tr>
<td>1</td>
<td>T&lt;sub&gt;D&lt;/sub&gt;</td>
<td>M→M + au</td>
</tr>
<tr>
<td></td>
<td>R&lt;sub&gt;u&lt;/sub&gt;</td>
<td>M→N + exp (fuA) NM</td>
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<tr>
<td></td>
<td>H&lt;sub&gt;u,c&lt;/sub&gt;</td>
<td>M→N + fu + exp (fuA) NM</td>
</tr>
<tr>
<td>2</td>
<td>T&lt;sub&gt;P&lt;/sub&gt;</td>
<td>M→M + au + bv</td>
</tr>
<tr>
<td></td>
<td>C&lt;sub&gt;u&lt;/sub&gt;</td>
<td>M→N + au + exp (fuA) NM</td>
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<tr>
<td>3</td>
<td>T</td>
<td>M→M + au + bv + cw</td>
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<tr>
<td></td>
<td>G&lt;sub&gt;P&lt;/sub&gt;</td>
<td>M→N + au + bw + exp (hwA) NM</td>
</tr>
<tr>
<td></td>
<td>Y&lt;sub&gt;u,c&lt;/sub&gt;</td>
<td>M→N + au + bw + khw + exp (hwA) NM</td>
</tr>
<tr>
<td>4</td>
<td>X&lt;sub&gt;u&lt;/sub&gt;</td>
<td>M→N + exp ((fu + gwv + hvw)A) NM</td>
</tr>
<tr>
<td>6</td>
<td>D</td>
<td>M→N + au + bv + cw + exp ((fu + gwv + hwA)A) NM</td>
</tr>
</tbody>
</table>

N, u, v, w represents an orthonormal direct frame of reference.

### Synoptic List 1

![Image](image-url)