Diversity-multiplexing tradeoffs in cooperative protocols with relay selection

Aggelos Bletsas and Ashish Khisti
Massachusetts Institute of Technology
77 Massachusetts Ave, Cambridge, MA 02139
aggelos@media.mit.edu, khisti@mit.edu

***Abstract*— We study the performance in cooperative diversity protocols employing relay selection, utilizing the diversity-multiplexing tradeoff. We note that for the class of orthogonal cooperation protocols studied by Laneman and Wornell, there is no loss in performance if only one (suitably chosen) relay participates in cooperation. This observation also holds in systems which incorporate decision feedback from the destination. We also quantify performance loss due to relay selection in non-orthogonal dynamic-decode-and-forward protocol studied by Azarian, El Gamal and Schniter. Our results present a simple alternative to distributed space-time codes for realizing the potential gains in multiple relay, user cooperation systems, and open new avenues for fruitful interaction between routing and cooperative diversity.**

I. Introduction

Cooperative diversity is an attractive approach to improve performance in slow fading environments by creating distributed virtual antennas across different nodes in the network. There has been a significant interest understanding the theoretical limits of such systems (e.g. [8],[9],[7],[1]) and therefore the study of practical architectures that achieve some of these limits is a fertile area of research.

Laneman et.al.[7] study a three node system with one source, one destination and one relay and propose several architectures which are amenable to practical implementation. The performance is characterized using the diversity-multiplexing tradeoff, an analytic tool proposed by Zheng and Tse [11] for multi-input-multi-output (MIMO) systems. Subsequently, Laneman and Wornell [8] have generalized the three node scenario to the case when multiple relay nodes help the source and destination. These architectures require that relays successful in decoding the source transmission help the destination by employing a distributed space-time code. While the performance of such systems increases monotonically with the number of relays, these architectures assume availability of optimal distributed space-time codes across the relay nodes. While there has been some progress towards constructing space-time codes for the MIMO systems (e.g. [12]), and cooperative systems involving a single relay (e.g. [13]) and multiple relay systems involving global CSI knowledge (e.g. [14]), it remains to be seen how such constructions can be generalized in practical networks involving multiple relays and low complexity radio front ends.

One architecturally appealing approach in multiple relay systems is relay selection. Based on end-to-end channel conditions, only one relay is used for cooperation. The system then reduces to the single-relay case for which several simple protocols exist. While such relay selection schemes cannot in principle outperform the protocols in [8] (that allow for simultaneous transmissions across the relays), we show a rather surprising result that with a proper choice of relay selection criterion there is in fact no performance loss in the sense of diversity-multiplexing tradeoff. While relay selection has been considered recently by other authors, to the best of our knowledge our work is the first to compute the full diversity-multiplexing tradeoff of such systems and show that there is no loss compared to architectures based on distributed space-time codes. We refer to our relay selection protocols as *opportunistic relaying*.

We observe that our relay selection protocols achieve the same diversity-multiplexing tradeoff in [8] irrespective of whether the relay nodes do amplify-and-forward (AF) or decode-and-forward (DF). Further, we extend our result to the case where decision feedback is available from the destination and we observe additional gains that increase with the number of rounds of feedback. Our work sheds new insights into low-complexity practical architectures that combine relay selection and cooperative diversity, without sacrificing system performance.

More recently it has been shown that simultaneous transmission across the source and relay node(s) can achieve better performance (e.g. [10],[1]). We note that relay selection protocols incur a loss when simultaneous transmissions are allowed and quantify the loss for the dynamic-decode-and-forward protocols in [1]. However, we also emphasize that simultaneous transmissions in the same frequency band from *distributed* terminals, require non-conventional transceiver designs and remain to be tested in practice.

II. Channel Model

In this section we describe the channel model under consideration. We consider a system with one source node, one destination node and M relay nodes. The channel gain between the source node and relay node i is $a_{\mathrm{s}i}$, between the relay node i and the destination is $a_{i\mathrm{d}}$ and between the source and destination is a_{sd}. We assume that all channel gains are drawn i.i.d. $\mathcal{CN}(0,1)$ and remain constant throughout the course of transmission, as in the slow fading (quasi-static) channel model. When the source transmits a symbol x_s, the received symbols at relay r and the destination are given by:

$$\begin{aligned} y_r &= a_{\mathrm{s}r}\ x_{\mathrm{s}} + n_r \\ y_{\mathrm{d}} &= a_{\mathrm{sd}}\ x_{\mathrm{s}} + n_{\mathrm{d}} \end{aligned} \tag{1}$$

Here n_r and n_{d} represent additive noise at the relay and destination respectively. We assume that these are complex Gaussian drawn independently from $\mathcal{CN}(0,N_0)$. Similarly, when the relay transmits a symbol x_r, the received symbol at the destination is given by:

$$y_{\mathrm{d}} = a_{r\mathrm{d}}\ x_r + n_{\mathrm{d}} \tag{2}$$

We shall assume an individual power constraint at each of the node. In particular $E[|X_s|^2] \leq P$ and $E[|X_r|^2] \leq P$ for $r = 1,2,\ldots,M$. Throughout we refer to $\rho \stackrel{\Delta}{=} P/N_0$ as the average SNR on each link. Finally, note that in the high SNR regime of interest, neither the individual power constraint nor the assumption of equal average SNR at each node are crucial towards our conclusions.

III. Protocols with or without Feedback

The protocols we consider have three main phases: (1) Distributed relay selection. (2) Transmission from the source and reception by the relay and the destination. (3) Transmission from the relay and reception by the destination. The three phases occur in the order as stated and over orthogonal channel uses.

a) Relay Selection: Throughout this work, we assume that the relay-selection is done before the transmission of message from the source commences. For a discussion on selection schemes after the source transmission concludes please refer to the full paper [5]. We consider two possible objective functions in the process of selecting the best relay with channel gains $|a_{\mathrm{sb}}|$ and $|a_{\mathrm{bd}}|$:

$$\min(|a_{\mathrm{sb}}|^2, |a_{\mathrm{bd}}|^2) = \max_{1\leq r\leq M}\{\min(|a_{\mathrm{s}r}|^2, |a_{r\mathrm{d}}|^2)\} \tag{3}$$

$$\frac{2\ |a_{\mathrm{sb}}|2\ |a_{\mathrm{bd}}|2}{|a_{\mathrm{sb}}|2 + |a_{\mathrm{bd}}|2} = \max_{1\leq r\leq M} \frac{2\ |a_{\mathrm{s}r}|2\ |a_{r\mathrm{d}}|2}{|a_{\mathrm{s}r}|2 + |a_{r\mathrm{d}}|2}. \tag{4}$$

It turns out that both of these choices provide the same tradeoff performance and hence, are equivalent.

b) Transmission from Source and Relay: : During the source transmission phase, the source transmits and the relay and destination listen to the transmission. During the relay transmission phase, the relay transmits using either amplify-and-forward or decode-and-forward protocols. If decision feedback is allowed, the relay only transmits if it receives a NACK (Negative Acknowledgement) from the destination, indicating failure of reception. In that sense, the "best" available relay is used only when necessary.

IV. Results

We first consider the setup of Laneman and Wornell where the source transmits in first half slots and the relay transmits in next half slots. We refer to these as orthogonal cooperation protocols.

Theorem 1: The diversity-multiplexing gain of zero-feedback opportunistic and DF relaying as well as AF relaying is:

$$d(r) = (M+1)(1-2r) \tag{5}$$

Note that the diversity-multiplexing tradeoff in the above Theorem is same as that for the M relay case studied by Laneman [8], using space-time codes. Our result shows that one can get essentially the same performance via opportunistic relaying and using either analog or digital relays. Thus the gains from user cooperation arise fundamentally from the existence of multiple paths rather than the use of distributed space-time codes. Note that there has been a lot of interest in relay selection algorithms for cooperative communications. To the best of our knowledge, our result is the first one that makes a precise statement regarding the diversity-multiplexing performance optimality of relay selection protocols.

We next note that similar gains are also possible if a decision feedback is available from the destination. In this case, the relay will transmit only if the destination fails to decode the source transmission.

Theorem 2: The diversity-multiplexing tradeoff of both DF and AF protocols with a single-bit feedback is given by $d(r) = (M+1)(1-r)$.

We note that a single-bit decision feedback can provide the performance of a classical $(M+1) \times 1$ MISO system with collocated antennas, which is substantially better than the case without feedback in Theorem 1. Even further gains are possible if multiple rounds of feedback are permitted.

Theorem 3: The achievable diversity-multiplexing tradeoff for AF protocols with L rounds of feedback (i.e. 2L-1 NACKs) is given by $d(r) = (M+1)(1-\frac{r}{L})$ for $0 \leq r \leq 1$.
A similar result has been observed for the MISO systems in [15] and for cooperative diversity systems employing dynamic DF strategies in [2].

More recently, non-orthogonal relay transmission has been proposed in [1] for improving the spectral efficiency. The source transmits over all available slots. Whenever the relay is successful in decoding the message, it switches into transmitting mode and starts cooperating with the source. This protocol is called dynamic-decode-and-forward. The diversity-multiplexing tradeoff of such protocols under relay selection suffers a loss. We note however that opportunistic relaying is simpler and can be implemented with existing radio front ends.

Theorem 4: The diversity-multiplexing tradeoff of dynamic-decode-and-forward under relay selection is given by:

$$d(r) = \begin{cases} (M+1)(1-r) & 0 \leq r \leq \frac{1}{M+1} \\ 1 + (M+1)(1-2r) & \frac{1}{M+1} \leq r \leq 1/2 \\ \frac{1-r}{r} & 1/2 \leq r \leq 1 \end{cases} \quad (6)$$

V. Conclusion

We explored the fruitful interaction between relay selection (which can be viewed as a routing protocol) and cooperative diversity (which is a physical layer protocol).

We hope that our work sparks further interest in implementation of distributed relay selection protocols. While one such protocol based on distributed timers has been proposed in [4] and the experimental results have been reported in [3], we believe this is a very fertile area for further innovation.

VI. Acknowledgements

Aggelos Bletsas would like to thank Prof. Moe Z. Win for his helpful comments throughout this work.

References

[1] K. Azarian, H. E. Gamal, and P. Schniter, "On the Achievable Diversity-vs-multiplexing Tradeoff in Cooperative Channels", *IEEE Trans. Information Theory*, vol. 51, pp. 4152-4172, Dec. 2005.

[2] K. Azarian, H. El Gamal and P. Schniter, "On the Achievable Diversity-Multiplexing Tradeoff in Half Duplex Cooperative Channels," Proc. Allerton Conf. on Communication, Control, and Computing, Oct. 2004, Monticello, IL.

[3] A. Bletsas, *Intelligent Antenna Sharing in Cooperative Diversity Wireless Networks*, Ph.D. Dissertation, Massachusetts Institute of Technology, September 2005.

[4] A. Bletsas, A. Khisti, D.P. Reed, A. Lippman, "A Simple Cooperative Diversity Method based on Network Path Selection", IEEE Journal on Selected Areas of Communication, Special Issue on 4G, submitted January 2005, accepted for publication, to appear. Available at `http://web.media.mit.edu/~aggelos/papers/revised_4G101.pdf`

[5] A. Bletsas, A. Khisti, M. Z. Win, "Unifying Cooperative Diversity and Routing with Select and Forward Protocols", In preparation.

[6] A. Bletsas, A. Lippman, D.P. Reed, "A Simple Distributed Method for Relay Selection in Cooperative Diversity Wireless Networks, based on Reciprocity and Channel Measurements", Proceedings of IEEE 61st VTC, May 30 - June 1 2005, Stockholm, Sweden.

[7] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative Diversity in Wireless Networks: Efficient Protocols and Outage Behavior," IEEE Trans. Inform. Theory, Accepted for publication, June 2004.

[8] J. N. Laneman and G. W. Wornell, "Distributed Space-Time Coded Protocols for Exploiting Cooperative Diversity in Wireless Networks," IEEE Trans. Inform. Theory, vol. 59, pp. 2415-2525, October 2003.

[9] A. Sendonaris, E. Erkip and B. Aazhang. User cooperation diversity-Part I: System description. IEEE Transactions on Communications, vol. 51, no. 11, pp. 1927-1938, November 2003.

[10] R. Nabar, H. Bolcskei and F. Kneubuhler, "Fading relay channels: performance limits and space-time signal design," IEEE Journal on Selected Areas in Communications, Aug. 2004 Vol. 22, Issue 6, pp 1099-1109

[11] L. Zheng and D. Tse, "Diversity and Multiplexing: A Fundamental Tradeoff in Multiple Antenna channels," *IEEE Trans. Inform. Theory*, vol. 49, pp. 1073–96, May, 2003.

[12] H.E.Gamal, G. Caire, M.O. Damen, "Lattice coding and decoding achieve the optimal diversity-multiplexing tradeoff of MIMO channels," pp. 968- 985, June 2004.

[13] M. Dohler, E. Lefranc and H. Aghvami "Space-Time Block Codes," for Virtual Antenna Arrays, PIMRC, Lisbon, Portugal, Sept 2002.

[14] Y. Jing and B. Hassibi, "Distributed space-time coding in wireless relay networks-Part I: basic diversity results", Submitted to IEEE Trans. On Wireless Communications, July 2004.

[15] H.E.Gamal, G. Caire, M.O. Damen, "The MIMO ARQ Channel," submitted IT Trans. Information Theory