Outage-Optimal Cooperative Communications with Regenerative Relays

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***Abstract*— In this paper, we put forth simple *opportunistic* relaying with decode-and-forward (DaF) or regenerative processing strategy, under an aggregate power constraint. In particular, we consider distributed relay-selection algorithms requiring only local channel knowledge. We prove that opportunistic single relay selection with DaF strategy is outage-optimal, that is, it is equivalent in outage behavior to the optimal DaF strategy that employs all potential relays.**

***Index Terms*— Cooperative diversity, fading relay channel, outage probability, wireless networks.**

I. Introduction

Significant improvements in the performance of wireless networks can be made by employing terminals distributed in space [1]–[3]. Basic results for single-relay cooperation are presented in [4], [5] and references therein. Although interests in cooperative communication have been immensely increased recently, scaling cooperation to more than one relay is still an open area of research.

Distributed space-time coding was recently proposed for multiple-relay scenarios [6]. However, the number of *useful* antennas (distributed relays) for cooperation is generally unknown and time-varying. Additional difficulties arise from the lack of global channel state information (CSI) in distributed environments. For example it is difficult in practice for the destination to acquire CSI between the source and *all* relays, as needed in the scheme proposed in [7]. Furthermore, *phased-array* techniques devised for co-located multi-antenna transmitters can not be easily applied to distributed multiple-relay (MR) transmissions. As a result, the superposition of MR transmissions at the same time and frequency can not be assumed as *always constructive*. Note that the case of always constructive addition includes Gaussian relay channels where propagation coefficients are assumed to be real numbers [8]. Finally, *coherent* reception of MR transmission requires significant overhead due to tracking of carrier-phase differences between multiple relays and destination.

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Therefore, simplification of cooperative communication techniques is crucial. Antenna selection, invented for classical multiple-antenna communications [9]–[13], is one approach to minimize the required *cooperation overhead* and to simultaneously realize the potential benefits of cooperation between multiple relays. In particular, a simple, distributed, single-relay selection algorithm was proposed for slow fading wireless relay channels [14], which are fundamentally different than co-located multi-antenna links. This single-relay *opportunistic* selection provides no performance loss from the perspective of diversity–multiplexing gain tradeoff, compared to schemes that rely on distributed space–time coding [15].

In this paper, we consider both *reactive* and *proactive* relay selection depending on whether the relay selection is performed after or before the source transmission. We prove that, under an aggregate power constraint, both reactive and proactive opportunistic relay selection with DaF processing strategy is outage-optimal, that is, they are equivalent in outage behavior to the optimal DaF strategy that employs all potential relays.

The remainder of the paper is organized as follows. In Section II, we present the basic protocols examined in this work and in Sections III, IV we present analysis for the reactive and proactive case, respectively. Section V presents the analytical results verified by Monte-Carlo simulations. Finally, conclusions are given in section VI.

II. Models and Protocols

We consider a half-duplex dual-hop communication scenario in a cluttered environment, where the direct path between the source and destination is blocked, while relays are located at the periphery of the obstacle. The DaF relays can communicate with both endpoints (source and destination). During the first phase, the source (without exploiting any CSI) transmits $N/2$ symbols and the relays listen, while during the second phase, the relays forward a version of the received signal using the same number of symbols.[1] The channel is assumed to remain constant during the two phases (at least N-symbol coherence time) with Rayleigh fading. We further consider a source power constraint and an aggregate relay

[1] If the source is allowed to transmit different symbols during the second hop, one channel degree of freedom would not be wasted and the spectral efficiency can be improved [4], [16]. However, in this paper, we are interested in finding the optimal strategy for relay transmissions and hence, simplify their operation.

power constraint

$$\mathcal{P}_{\text{source}} = \zeta \mathcal{P}_{\text{tot}}, \quad \mathcal{P}_{\text{relay}} = \sum_{k=1}^{K} \mathcal{P}_k = (1-\zeta)\,\mathcal{P}_{\text{tot}}\,, \tag{1}$$

where K is the number of relays, $\mathcal{P}_{\text{tot}}$ is the total end-to-end (i.e., source-relay-destination) transmission power, $\mathcal{P}_{\text{source}}$ is the transmission power of the source, $\mathcal{P}_k$, $k = 1, \ldots, K$, is the transmission power of the kth relay, and $\mathcal{P}_{\text{relay}}$ is the aggregate relay power allocated to the set $\mathcal{S}_{\text{relay}} = \{1, 2, \ldots, K\}$ of K relays. Note that $\zeta \in (0, 1]$ and $(1-\zeta) \in [0, 1)$ denote the fractions of the total end-to-end power $\mathcal{P}_{\text{tot}}$ allocated to the source transmission and overall relay transmission, respectively.

It should be noted that the optimal power allocation across the source and relays depends on CSI knowledge and can be $\mathcal{P}_{\text{source}} \neq \mathcal{P}_{\text{relay}}$ [17]. However, this is feasible only when global CSI about the whole network (including channel states between the relays and destination) is available at the source. In this work, we do not assume CSI at the source.

The received signal in a link (A $\to$ B) between two nodes "A" and "B" is given by

$$y_{\text{B}} = \alpha_{\text{AB}}\, x_{\text{A}} + n_{\text{B}} \tag{2}$$

where x_{A} is the signal transmitted at the node A, $\alpha_{\text{AB}} \sim \mathcal{CN}(0, \Omega_{\text{AB}})$ is the channel gain between the link A $\to$ B, and $n_{\text{B}} \sim \mathcal{CN}(0, N_0)$ is the additive white Gaussian noise (AWGN) at the node B.[2] For each link, let $\gamma_{\text{AB}} \triangleq |\alpha_{\text{AB}}|^2$ be the instantaneous squared channel strength, which obeys an exponential distribution with hazard rate $1/\Omega_{\text{AB}}$, denoted by $\gamma_{\text{AB}} \sim \Upsilon(1/\Omega_{\text{AB}})$.

If the node A is the source, then $\mathbb{E}\{|x_{\text{A}}|^2\} = \mathcal{P}_{\text{source}}$. Similarly, if the node A is the kth relay, then $\mathbb{E}\{|x_{\text{A}}|^2\} = \mathcal{P}_k$. Specifically, for each relay $k \in \mathcal{S}_{\text{relay}}$, we designate a link from the source to the kth relay by S $\to k$ and a link from the kth relay to the destination by $k \to$ D. Also, R denotes the end-to-end spectral efficiency in bps/Hz and $\mathsf{SNR} \triangleq \mathcal{P}_{\text{tot}}/N_0$ denotes the end-to-end transmit signal-to-noise ratio (SNR). For the links S $\to k$ and $k \to$ D, the average received SNRs are equal to $\eta_{\text{S}k} \triangleq \Omega_{\text{S}k}\mathcal{P}_{\text{source}}/N_0$ and $\eta_{k\text{D}} \triangleq \Omega_{k\text{D}}\mathcal{P}_k/N_0$, respectively.[3]

To reduce overhead and simplify protocol implementation, cooperation is coordinated only every N symbols. We consider both *reactive* and *proactive* coordination among the relays. In reactive mode, relays that successfully decode the message participate in cooperation, whereas in a proactive mode, specific relays are selected prior to the source transmission.

Relay selection can be performed without requiring global CSI at each relay or a central controller in the network. One possible approach is to use the method of distributed timers proposed in [14], where each relay estimates its own instantaneous channel paths towards source and destination. This can be accomplished by listening pilot signals from source and destination. Upon receiving a pilot signal from the destination, each relay then starts a timer with duration proportional to a function that depends only on its *own* channel gains towards source and destination. The timer of the "best" relay expires first and a broadcast packet notifies the rest of the network about its availability. The relay selection is completed within a fraction of the channel coherence time and the selected single relay is then used for information relaying. Statistical analysis of such protocol was reported in [15].

III. Reactive DaF

A. Reactive Multiple-Relay DaF

In a reactive MR scheme with DaF strategy, the relays that successfully receive the message during the first phase, regenerate and transmit it during the second phase, possibly through a distributed space–time code [6]. The transmissions, during the second phase, are performed only by a subset $\mathcal{D}$ of K relays, defined by

$$\mathcal{D} \triangleq \left\{ k \in \mathcal{S}_{\text{relay}} : \frac{1}{2}\log_2\left(1 + \zeta \mathsf{SNR} \cdot \gamma_{\text{S}k}\right) \geq R \right\} \tag{3}$$

where the decoding at relay k is assumed to be successful if $\frac{1}{2}\log_2\left(1 + \zeta \mathsf{SNR} \cdot \gamma_{\text{S}k}\right) \geq R$, i.e., no outage event happens during the first phase [3], [6]. Since communication happens in two half-duplex hops, the required spectral efficiency per hop is equal to $2R$ so that the end-to-end spectral efficiency is R, which is comparable to direct non-cooperative communication.

Let $\mathcal{D}_\ell \subseteq \mathcal{S}_{\text{relay}}$ be a decoding subset with ℓ relays (i.e., cardinality $|\mathcal{D}_\ell| = \ell$). Then, we have

$$\begin{aligned} \Pr\{\mathcal{D}_\ell\} &= \prod_{i \in \mathcal{D}_\ell} \Pr\{\gamma_{\text{S}i} \geq \kappa_1\} \prod_{j \notin \mathcal{D}_\ell} \Pr\{\gamma_{\text{S}j} \leq \kappa_1\} \\ &= \prod_{i \in \mathcal{D}_\ell} e^{-\frac{\kappa_1}{\Omega_{\text{S}i}}} \prod_{j \notin \mathcal{D}_\ell} \left(1 - e^{-\frac{\kappa_1}{\Omega_{\text{S}j}}}\right) \end{aligned} \tag{4}$$

where $\kappa_1 = \frac{2^{2R}-1}{\zeta \mathsf{SNR}}$. The outage probability for reactive MR transmissions with DaF strategy can be written as

$$P_{\text{MR}}^{(\text{react})}(\text{outage}) = \sum_{\ell=0}^{K} \sum_{\mathcal{D}_\ell} \Pr\Big\{ \text{outage} \big| \, \mathcal{D}_\ell \Big\} \Pr\{\mathcal{D}_\ell\} \tag{5}$$

where the second summation is over all $\binom{K}{\ell}$ different decoding subsets with exactly ℓ successfully decodable relays. The equality in (5) is due to the total probability theorem over disjoint sets $\mathcal{D}_\ell$ that partition the sample space. Note that there are 2^K possible decoding subsets for K relays, including $\mathcal{D}_0$, i.e., the set with no decodable relay during the first hop of the protocol. In (5), the conditional outage probability is given by

$$\Pr\Big\{ \text{outage} \big| \, \mathcal{D}_\ell \Big\} = \Pr\left\{ \frac{1}{2}\log_2\left(1 + \sum_{k \in \mathcal{D}_\ell} \frac{\mathcal{P}_k \gamma_{k\text{D}}}{N_0}\right) < R \right\}, \tag{6}$$

with $\sum_{k \in \mathcal{D}_\ell} \mathcal{P}_k = \mathcal{P}_{\text{relay}}$.

Let $\{\varphi_i(\mathcal{D}_\ell)\}_{i=1}^{\ell} = \{\eta_{k\text{D}}\}_{k \in \mathcal{D}_\ell}$ and $\boldsymbol{\mathcal{A}}(\mathcal{D}_\ell) = \operatorname{diag}\left(\varphi_1(\mathcal{D}_\ell), \varphi_2(\mathcal{D}_\ell), \ldots, \varphi_\ell(\mathcal{D}_\ell)\right)$. Then, it follows from

[2] $\mathcal{CN}(\mu, \sigma^2)$ denotes a complex circularly symmetric Gaussian distribution with mean μ and variance σ^2. Similarly, $\tilde{\mathcal{N}}_m(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ denotes a complex m-variate Gaussian distribution with a mean vector $\boldsymbol{\mu} \in \mathbb{C}^m$ and a covariance matrix $\boldsymbol{\Sigma} \in \mathbb{C}^{m \times m}$.

[3] We consider a scenario in which the channel gains for all links are statistically independent and all terminals have equal average noise power.

Theorem 1 in Appendix that

$$\Pr\Big\{\text{outage}|\,\mathcal{D}_\ell\Big\} = 1 - \sum_{i=1}^{\varrho(\boldsymbol{\mathcal{A}}(\mathcal{D}_\ell))}\sum_{j=1}^{\vartheta_i(\boldsymbol{\mathcal{A}}(\mathcal{D}_\ell))}\sum_{k=0}^{j-1} e^{-\frac{2^{2R}-1}{\varphi_{\langle i\rangle}(\mathcal{D}_\ell)}} \times \frac{\mathcal{X}_{i,j}\left(\boldsymbol{\mathcal{A}}\left(\mathcal{D}_\ell\right)\right)}{k!}\left(\frac{2^{2R}-1}{\varphi_{\langle i\rangle}\left(\mathcal{D}_\ell\right)}\right)^k, \quad (7)$$

where $\varrho\left(\boldsymbol{\mathcal{A}}\left(\mathcal{D}_\ell\right)\right)$ is the number of distinct diagonal elements of $\boldsymbol{\mathcal{A}}\left(\mathcal{D}_\ell\right)$, $\varphi_{\langle 1\rangle}\left(\mathcal{D}_\ell\right) > \varphi_{\langle 2\rangle}\left(\mathcal{D}_\ell\right) > \ldots > \varphi_{\langle \varrho(\boldsymbol{\mathcal{A}}(\mathcal{D}_\ell))\rangle}\left(\mathcal{D}_\ell\right)$ are the distinct diagonal elements in decreasing order, $\vartheta_i\left(\boldsymbol{\mathcal{A}}\left(\mathcal{D}_\ell\right)\right)$ is the multiplicity of $\varphi_{\langle i\rangle}\left(\mathcal{D}_\ell\right)$, and $\mathcal{X}_{i,j}\left(\boldsymbol{\mathcal{A}}\left(\mathcal{D}_\ell\right)\right)$ is the (i,j)th characteristic coefficient of $\boldsymbol{\mathcal{A}}\left(\mathcal{D}_\ell\right)$ [18, Definition 6]. Combining (4), (5), and (7) gives

$$\begin{aligned} &P_{\text{MR}}^{(\text{react})}\left(\text{outage}\right) \quad (8)\\ &= \sum_{\ell=0}^{K}\sum_{\mathcal{D}_\ell}\Bigg\{\prod_{i\in\mathcal{D}_\ell} e^{-\frac{\kappa_1}{\Omega_{\text{S}i}}}\prod_{j\notin\mathcal{D}_\ell}\left(1-e^{-\frac{\kappa_1}{\Omega_{\text{S}j}}}\right)\\ &\quad\times\Bigg[1-\sum_{i=1}^{\varrho(\boldsymbol{\mathcal{A}}(\mathcal{D}_\ell))}\sum_{j=1}^{\vartheta_i(\boldsymbol{\mathcal{A}}(\mathcal{D}_\ell))}\sum_{k=0}^{j-1}\frac{\mathcal{X}_{i,j}\left(\boldsymbol{\mathcal{A}}\left(\mathcal{D}_\ell\right)\right)}{k!}\\ &\quad\times\left(\frac{2^{2R}-1}{\varphi_{\langle i\rangle}\left(\mathcal{D}_\ell\right)}\right)^k e^{-\frac{2^{2R}-1}{\varphi_{\langle i\rangle}(\mathcal{D}_\ell)}}\Bigg]\Bigg\}. \end{aligned}$$

B. Reactive Opportunistic DaF

For opportunistic relaying, the "best" relay b^* among the ℓ relays in the decoding subset $\mathcal{D}_\ell$ is chosen naturally to maximize the *instantaneous* channel strength between the links $k \to \text{D}$ for all $k \in \mathcal{D}_\ell$:

$$b^* = \arg\max_{k\in\mathcal{D}_\ell}\gamma_{k\text{D}}\,. \quad (9)$$

Therefore, the receive SNR at the destination satisfies

$$\sum_{k\in\mathcal{D}_\ell}\frac{\mathcal{P}_k\gamma_{k\text{D}}}{N_0} \le \sum_{k\in\mathcal{D}_\ell}\frac{\mathcal{P}_k\gamma_{b^*\text{D}}}{N_0} = \gamma_{b^*\text{D}}\cdot(1-\zeta)\,\mathsf{SNR} \quad (10)$$

and this conditional outage probability in (6) is lower bounded by

$$\begin{aligned} &\Pr\Big\{\text{outage}|\,\mathcal{D}_\ell\Big\}\\ &\ge \Pr\left\{\frac{1}{2}\log_2\Big(1+(1-\zeta)\,\mathsf{SNR}\max_{k\in\mathcal{D}_\ell}\gamma_{k\text{D}}\Big) < R\right\}\\ &= \prod_{k\in\mathcal{D}_\ell}\Pr\left\{\gamma_{k\text{D}}<\kappa_2\right\} \quad (11)\end{aligned}$$

where $\kappa_2 = \frac{2^{2R}-1}{(1-\zeta)\mathsf{SNR}}$. Note that (11) states simply that if the "best" relay fails, then all relays in $\mathcal{D}_\ell$ fail because the "best" relay has the strongest path $\gamma_{b^*\text{D}}$ between the links $k \to \text{D}$ for all $k \in \mathcal{D}_\ell$. The maximum receive SNR in (10) and the minimum conditional outage probability in (11) are both satisfied by the opportunistic relay-selection rule (9). We remark that the minimization of (11) holds for any power allocation ζ. For quasi-static fading environments, a simple method can be devised to select the relay with the maximum channel strength $\gamma_{b^*\text{D}}$ in a distributed manner similar to the work in [14], [15].

Using (4) and (5) in conjunction with (11) for the opportunistic relay-selection rule (9), we obtain the outage probability for reactive opportunistic DaF relaying as

$$\begin{aligned} &P_{\text{Opp}}^{(\text{react})}\left(\text{outage}\right)\\ &= \sum_{\ell=0}^{K}\sum_{\mathcal{D}_\ell}\left[\prod_{i\in\mathcal{D}_\ell}e^{-\frac{\kappa_1}{\Omega_{\text{S}i}}}\left(1-e^{-\frac{\kappa_2}{\Omega_{i\text{D}}}}\right)\prod_{j\notin\mathcal{D}_\ell}\left(1-e^{-\frac{\kappa_1}{\Omega_{\text{S}j}}}\right)\right]\\ &= \prod_{k=1}^{K}\left[1-e^{-\frac{2^{2R}-1}{\mathsf{SNR}}\left(\frac{1}{\zeta\Omega_{\text{S}k}}+\frac{1}{(1-\zeta)\Omega_{k\text{D}}}\right)}\right] \quad (12)\end{aligned}$$

where the last equality follows from the multinomial equality

$$\begin{aligned}\prod_{k=1}^{K}\left(1-a_k b_k\right) &= \prod_{k=1}^{K}\Big[a_k\left(1-b_k\right)+\left(1-a_k\right)\Big] \quad (13)\\ &= \sum_{\ell=0}^{K}\sum_{\substack{\mathcal{S}_\ell\subseteq\{1,2,\ldots,K\}\\|\mathcal{S}_\ell|=\ell}}\left[\prod_{i\in\mathcal{S}_\ell}a_i\left(1-b_i\right)\prod_{j\notin\mathcal{S}_\ell}\left(1-a_i\right)\right].\end{aligned}$$

Note that (12) implies that opportunistic DaF relaying is in outage only when all of the relays are in outage. In this case, no other schemes can communicate reliably at rate R. Hence, the reactive opportunistic DaF relaying is optimal under the aggregate relay power constraint (1) in a sense that it minimizes the end-to-end outage probability.

In contrast to our single-relay opportunistic rule, one may consider selection of the relay that maximizes the *average* forward channel gain among the decoding set:

$$b^* = \arg\max_{k\in\mathcal{D}_\ell}\mathbb{E}\left\{\gamma_{k\text{D}}\right\} = \arg\max_{k\in\mathcal{D}_\ell}\Omega_{k\text{D}}\,. \quad (14)$$

Such a choice was proposed in [19], [20].

IV. PROACTIVE DAF

It might seem that selecting a single relay *before* information is transmitted from the source, could potentially result in degraded performance. On the other hand, selecting a single relay for information forwarding simplifies the receiver design and the overall network operation, since proactive selection is equivalent to *routing*. In the following section, we show that such choice on protocol design incurs no performance loss.

In proactive opportunistic relaying, the "best" relay b^* is chosen prior to the source transmission among a collection of K possible candidates in a distributed fashion that requires each relay to know its own instantaneous signal strength (but not phase) between the links $\text{S} \to k$ and $k \to \text{D}$, for each relay $k \in \mathcal{S}_{\text{relay}}$.[4] The "best" relay b^* is chosen to maximize the minimum of the weighted channel strengths between the links $\text{S} \to k$ and $k \to \text{D}$ for all $k \in \mathcal{S}_{\text{relay}}$:[5]

$$b^* = \arg\max_{k\in\mathcal{S}_{\text{relay}}} W_k \quad (15)$$

[4]Relay selection can be accomplished using a method of distributed timers described in section II, without requiring global CSI.

[5]Instead of the minimum, the harmonic mean of two path strengths has been also considered in [14].

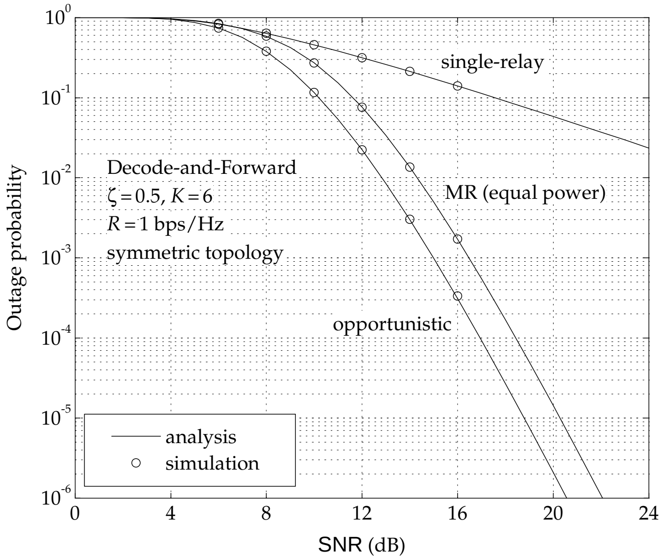

Fig. 1. Outage probability as a function of SNR for the DaF strategy at the end-to-end spectral efficiency $R = 1$ bps/Hz in symmetric channels. $\zeta = 0.5$, $K = 6$, and $\Omega_{\mathrm{S}k} = \Omega_{k\mathrm{D}} = 1$, $k = 1, 2, \ldots, 6$.

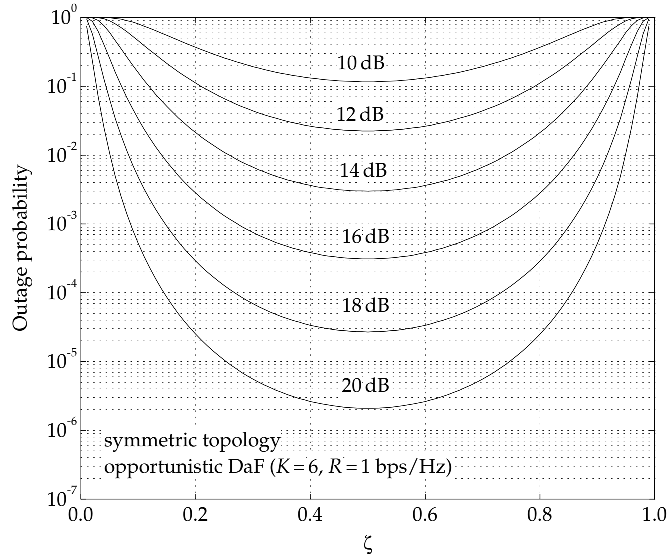

Fig. 2. Outage probability as a function of power allocation ζ for opportunistic DaF relaying at the end-to-end spectral efficiency $R = 1$ bps/Hz in symmetric channels when SNR = 10, 12, 14, 16, 18, and 20 dB. $K = 6$ and $\Omega_{\mathrm{S}k} = \Omega_{k\mathrm{D}} = 1$, $k = 1, 2, \ldots, 6$.

where $W_k = \min\{\zeta\gamma_{\mathrm{S}k}, (1-\zeta)\gamma_{k\mathrm{D}}\}$.

In this case, communication through the "best" opportunistic relay fails due to outage when either of the two hops (from the source to the best relay or from the best relay to destination) fail. Recall that

$$W_k \sim \Upsilon\left(\frac{1}{\zeta\Omega_{\mathrm{S}k}} + \frac{1}{(1-\zeta)\Omega_{k\mathrm{D}}}\right) \tag{16}$$

which follows from the fact that the minimum of two independent exponential random variables (r.v's) is again an exponential r.v. with a hazard rate equal to the sum of the two hazard rates. From (15) and (16), we obtain the outage probability for proactive opportunistic DaF relaying as follows:

$$\begin{aligned} P_{\text{Opp}}^{\text{proact}}(\text{outage}) &= \Pr\left\{W_{b^*} < \frac{2^{2R}-1}{\mathsf{SNR}}\right\} \\ &= \Pr\left\{\max_{k\in\mathcal{S}_{\text{relay}}} W_k < \frac{2^{2R}-1}{\mathsf{SNR}}\right\} \\ &= \prod_{k=1}^{K} \Pr\left\{W_k < \frac{2^{2R}-1}{\mathsf{SNR}}\right\} \\ &= \prod_{k=1}^{K}\left[1 - e^{-\frac{2^{2R}-1}{\mathsf{SNR}}\left(\frac{1}{\zeta\Omega_{\mathrm{S}k}} + \frac{1}{(1-\zeta)\Omega_{k\mathrm{D}}}\right)}\right]. \end{aligned} \tag{17}$$

It is worth remarking that the outage probability in (17) agrees *exactly* with that in (12) for reactive opportunistic relays with DaF strategy. Since we have shown in the previous section that the latter scheme is outage-optimal, the proactive opportunistic *"max-min"* relay selection in (15) is indeed outage-optimal. Moreover, proactive coordination requires a smaller cooperation overhead in reception energy since all relays, except a single opportunistic relay, can enter an idle mode during the first hop of the protocol. Therefore, our proactive strategy can be viewed as energy-efficient routing in the network. In contrast, the reactive schemes require all relays to receive information during the first hop and therefore, cooperation overhead in reception energy scales proportionally with the network size. This overhead may not be negligible especially in battery-operated terminals, when strong forward error correction (which requires energy-expensive routines) is used.

V. Numerical Results

In this section, we give numerical examples of the outage probability as a function of SNR with power allocation $\zeta = 0.5$. The optimal power allocation ζ is feasible, only when the source has knowledge of the overall network topology in terms of the average channel gains $\Omega_{\mathrm{S}k}$ and $\Omega_{k\mathrm{D}}$ for all participating relays $k \in \mathcal{S}_{\text{relay}}$. However, this is impractical since such knowledge requires considerable overhead. Therefore, the equal-power allocation to the source and the best opportunistic relay, i.e., $\zeta = 0.5$ is a natural choice. We further quantify the performance difference between $\zeta = 0.5$ and optimal choice of ζ that requires global CSI at the transmitter and the relay. Our results accommodate both symmetric and asymmetric topologies.

Fig. 1 shows the outage probability as a function of SNR for the DaF strategy with 6 relays ($K = 6$) at the end-to-end spectral efficiency $R = 1$ bps/Hz in symmetric channels with $\Omega_{\mathrm{S}k} = \Omega_{k\mathrm{D}} = 1$, $k = 1, 2, \ldots, 6$. In this figure, we show the performance of (i) proactive opportunistic DaF relaying, (ii) reactive DaF relaying with equal-power MR transmissions, and (iii) reactive DaF relaying via single-relay selection based on the maximum average channel gain $\max_{k\in\mathcal{D}_\ell}\Omega_{k\mathrm{D}}$. Fig. 2 plots outage probability as a function of power allocation ζ for the symmetric scenario and various SNR levels. Fig. 3 compares the same scenarios (as in Fig. 1) in asymmetric channels with $\{\Omega_{\mathrm{S}k}\}_{k=1}^{K} = \{\Omega_{k\mathrm{D}}\}_{k=1}^{K} = \{4.5, 0.5, 0.4, 0.3, 0.2, 0.1\}$. Finally, Fig. 4 plots outage probability as a function of ζ,

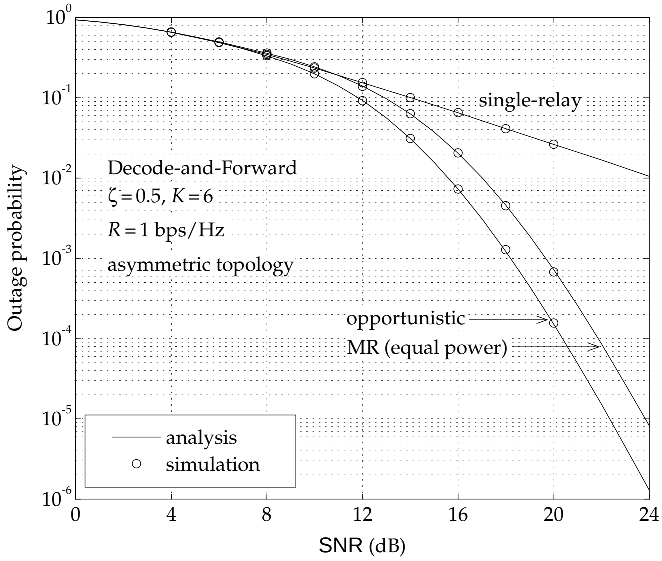

Fig. 3. Outage probability as a function of SNR for the DaF strategy at the end-to-end spectral efficiency $R = 1$ bps/Hz in asymmetric channels. $\zeta = 0.5$, $K = 6$, and $\{\Omega_{\mathrm{S}k}\}_{k=1}^{K} = \{\Omega_{k\mathrm{D}}\}_{k=1}^{K} = \{4.5, 0.5, 0.4, 0.3, 0.2, 0.1\}$.

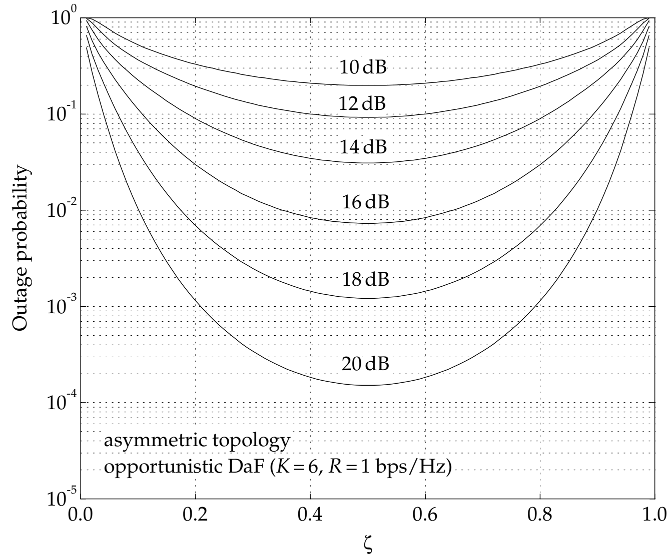

Fig. 4. Outage probability as a function of power allocation ζ for opportunistic DaF relaying at the end-to-end spectral efficiency $R = 1$ bps/Hz in asymmetric channels when SNR = 10, 12, 14, 16, 18, and 20 dB. $K = 6$ and $\{\Omega_{\mathrm{S}k}\}_{k=1}^{K} = \{\Omega_{k\mathrm{D}}\}_{k=1}^{K} = \{4.5, 0.5, 0.4, 0.3, 0.2, 0.1\}$.

for the asymmetric scenario. Note that for the symmetric case, single-relay selection based on the average channel gains amounts to selecting just one successful relay randomly (since all relays in the decoding subset $\mathcal{D}_\ell$ have the same average channel gain to the destination) and transmitting with full relaying power $\mathcal{P}_{\text{relay}}$. Note also that under limited channel knowledge at each relay, the optimal power allocation for MR transmission with DaF strategy is infeasible and equal power for the decoding subset $\mathcal{D}_\ell$, i.e., $\mathcal{P}_k = \mathcal{P}_{\text{relay}}/\ell$ for all $k \in \mathcal{D}_\ell$ is a reasonable solution, in reactive DaF relaying.

Both Figs. 1 and 3 show that opportunistic relaying, despite its simplicity, provides a gain in SNR on the order of 2 dB relative to MR transmission with DaF strategy. This finding reveals that cooperative diversity gains do not necessarily arise from simultaneous transmissions but instead, resilience to fading arises from the availability of several potential paths towards the destination. It is therefore beneficial to select the "best" one.[6] In contrast to single *opportunistic* relay selection, Figs. 1 and 3 also show that single-relay selection based on *average* channel gains incurs a substantial penalty loss. This is due to the fact that selecting a relay with average channel gains, in slow fading environments, removes potential selection diversity benefits.

Figs. 2 and 4 show that $\zeta = 0.5$ is optimal, for both symmetric and asymmetric scenarios, respectively. This can be verified analytically by differentiating (12) (or (17)). We note however that for the general case of $\Omega_{\mathrm{S}k} \neq \Omega_{k\mathrm{D}}$, for any relay k, optimal ζ^* will be different from 0.5.

[6]The main difficulty here is to have the network as a whole entity cooperate in order to rapidly discover the best path with minimal overhead. Ideas on how such selection can be performed in a distributed manner for slow fading environments can be found in [14], where actual implementations with low-cost radios were demonstrated.

VI. Conclusion

This paper presented *opportunistic* relaying protocols and analyzed outage performance under an aggregate power constraint. In particular, we proposed simple opportunistic relaying protocols that can be implemented in distributed manners without requiring global CSI. We proved that both reactive and proactive opportunistic DaF relaying are outage-optimal. Finally, we demonstrated that equal power allocation between the source and opportunistic relay gives near-optimal performance, without requiring global CSI.

Proactive opportunistic relaying allows all relays, except a single opportunistic relay, to enter an idle mode even during the source transmission, which reduces the reception energy cost in the network. Therefore, our proactive strategy can be viewed as energy-efficient routing in the network. In contrast, the reactive schemes require all relays to receive information during the source transmission and consequently, scale the reception energy proportionally with the network size. This overhead may not be negligible, especially in battery-operated terminals.

Our results reveal that relays in cooperative communications can be viewed not only as active re-transmitters, but also as distributed sensors of the wireless channel. Cooperative relays can be useful even when they do not transmit, provided that they cooperatively *listen*. In that way, cooperation benefits can be cultivated with simple radio implementation. An implementation example can be found in [14].

Appendix
Sum Distribution Statistics

Theorem 1 (Sum Distribution): Let $Y_n \sim \Upsilon(1/\mu_n)$, $n = 1, 2, \ldots, N$, be N statistically independent and not necessarily identically distributed (i.n.i.d.) exponential r.v.'s. Then, the p.d.f. of a sum $X = \sum_{n=1}^{N} Y_n$ for $x \geq 0$ is given by

$$p_X(x) = \sum_{i=1}^{\varrho(\boldsymbol{\mathcal{A}})} \sum_{j=1}^{\vartheta_i(\boldsymbol{\mathcal{A}})} \mathcal{X}_{i,j}(\boldsymbol{\mathcal{A}}) \frac{\mu_{\langle i \rangle}^{-j}}{(j-1)!} x^{j-1} e^{-x/\mu_{\langle i \rangle}}, \quad (18)$$

where $\boldsymbol{\mathcal{A}} = \text{diag}(\mu_1, \mu_2, \ldots, \mu_N)$, $\varrho(\boldsymbol{\mathcal{A}})$ is the number of distinct diagonal elements of $\boldsymbol{\mathcal{A}}$, $\mu_{\langle 1 \rangle} > \mu_{\langle 2 \rangle} > \ldots > \mu_{\langle \varrho(\boldsymbol{\mathcal{A}}) \rangle}$ are the distinct diagonal elements in decreasing order, $\vartheta_i(\boldsymbol{\mathcal{A}})$ is the multiplicity of $\mu_{\langle i \rangle}$, and $\mathcal{X}_{i,j}(\boldsymbol{\mathcal{A}})$ is the (i,j)th *characteristic coefficient* of $\boldsymbol{\mathcal{A}}$ [18, Definition 6]. The c.d.f. of X for $x \geq 0$ is given by

$$F_X(x) = 1 - \sum_{i=1}^{\varrho(\boldsymbol{\mathcal{A}})} \sum_{j=1}^{\vartheta_i(\boldsymbol{\mathcal{A}})} \sum_{k=0}^{j-1} \frac{\mathcal{X}_{i,j}(\boldsymbol{\mathcal{A}})}{k!} \left(\frac{x}{\mu_{\langle i \rangle}} \right)^k e^{-x/\mu_{\langle i \rangle}}. \quad (19)$$

Proof: Since $Y_1, Y_2, \ldots, Y_N$ are statistically independent, the characteristic function (c.f.) of X is

$$\Phi_X(\jmath w) \triangleq \mathbb{E}\left\{ e^{\jmath w X} \right\} = \prod_{n=1}^{N} (1 - \jmath w \mu_n)^{-1}. \quad (20)$$

Using a partial fraction decomposition of (20) with the characteristic coefficients, we obtain the p.d.f. of X as

$$\begin{aligned} p_X(x) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-\jmath w x} \Phi_X(\jmath w) \, dw \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{\jmath w x} \det(\mathbf{I}_N + \jmath w \boldsymbol{\mathcal{A}})^{-1} \, dw \\ &= \sum_{i=1}^{\varrho(\boldsymbol{\mathcal{A}})} \sum_{j=1}^{\vartheta_i(\boldsymbol{\mathcal{A}})} \frac{\mathcal{X}_{i,j}(\boldsymbol{\mathcal{A}})}{2\pi} \int_{-\infty}^{\infty} e^{\jmath w x} \left(1 + \jmath w \mu_{\langle i \rangle}\right)^{-j} dw \\ &= \sum_{i=1}^{\varrho(\boldsymbol{\mathcal{A}})} \sum_{j=1}^{\vartheta_i(\boldsymbol{\mathcal{A}})} \mathcal{X}_{i,j}(\boldsymbol{\mathcal{A}}) \frac{\mu_{\langle i \rangle}^{-j}}{(j-1)!} x^{j-1} e^{-x/\mu_{\langle i \rangle}} u(x) \quad (21) \end{aligned}$$

where $u(x)$ is the Heaviside step function.

From (21), we obtain the c.d.f. of X as

$$\begin{aligned} F_X(x) &= \sum_{i=1}^{\varrho(\mathbf{A})} \sum_{j=1}^{\vartheta_i(\mathbf{A})} \mathcal{X}_{i,j}(\mathbf{A}) \frac{\mu_{\langle i \rangle}^{-j}}{(j-1)!} \int_0^x t^{j-1} e^{-t/\mu_{\langle i \rangle}} dt \\ &= \sum_{i=1}^{\varrho(\mathbf{A})} \sum_{j=1}^{\vartheta_i(\mathbf{A})} \mathcal{X}_{i,j}(\mathbf{A}) \left[1 - \frac{1}{(j-1)!} \Gamma\left(j, \frac{x}{\mu_{\langle i \rangle}}\right) \right] \\ &= 1 - \sum_{i=1}^{\varrho(\mathbf{A})} \sum_{j=1}^{\vartheta_i(\mathbf{A})} \frac{\mathcal{X}_{i,j}(\mathbf{A})}{(j-1)!} \Gamma\left(j, \frac{x}{\mu_{\langle i \rangle}}\right) \quad (22) \end{aligned}$$

where the last equality follows from the fact that the sum of all the characteristic coefficients is equal to one [18], and $\Gamma(n, z)$ is the incomplete gamma function defined by

$$\Gamma(n, z) \triangleq \int_z^{\infty} t^{n-1} e^{-t} dt. \quad (23)$$

Finally, using the identity [21, eq. (8.352.2)]

$$\Gamma(n, z) = (n-1)! \, e^{-z} \sum_{k=0}^{n-1} \frac{z^k}{k!}, \text{ for an integer } n \quad (24)$$

yields the desired result (19). We remark that the above theorem can provide corollaries that agree with the well-known fact that a sum of N i.i.d. exponential r.v.'s has a central chi-squared distribution with $2N$ degrees of freedom.

□

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