Outage analysis for co-operative communication with multiple amplify-and-forward relays

A. Bletsas, H. Shin and M.Z. Win

Formation of virtual antenna arrays among neighbouring nodes and their co-operative transmission have attracted increasing interest. The exact outage probability of co-operative communication with multiple amplify-and-forward relays is given. The analysis covers general relay topologies and is verified by Monte-Carlo simulations.

Introduction and problem formulation: The area of co-operative communication, where neighbouring nodes co-operate by forming virtual antenna arrays, has attracted considerable interest [1–3]. This co-operative communication involves several distributed links with unequal path loss and hence radically deviates from a classical case of co-located multiple-antenna systems, making analysis difficult and providing a fertile area for further research. In this Letter, we present exact expressions for the outage probability of co-operative transmission using multiple amplify-and-forward (AF) relays in Rayleigh fading. Let the received signal in any link ($A \to B$) between two nodes 'A' and 'B' be

$$y_B = \alpha_{A,B} x_A + n_B \tag{1}$$

where x_A is the signal transmitted at node A, $\alpha_{A,B} \sim \mathcal{CN}(0, \Omega_{A,B})$ is the channel gain between the link $A \to B$, and $n_B \sim \mathcal{CN}(0, N_0)$ is the additive white Gaussian noise (AWGN) at node B. $\mathcal{CN}(\mu, \sigma^2)$ denotes a complex circularly symmetric Gaussian distribution with mean μ and variance σ^2. We consider a single source node S, destination node D, and K AF relays (see Fig. 1). If A is the source node, then $\mathbb{E}\{|x_A|^2\} \triangleq \mathcal{P}_S$ while $\mathbb{E}\{|x_k|^2 \triangleq \mathcal{P}_k$ for each relay $k \in \mathcal{S}_{relay} = \{1, 2, \ldots, K\}$. We further denote the end-to-end spectral efficiency in bit/s/Hz by R and the average received signal-to-noise ratios (SNRs) by $\Gamma_{S,k} = \Omega_{S,k}\mathcal{P}_k/N_0$ for the source-to-kth relay link and $\Gamma_{k,D} = \Omega_{k,D}\mathcal{P}_k/N_0$ for the kth relay-to-destination link. We consider a two-phase communication protocol where the source can communicate with the destination only through half-duplex AF relays (no direct path between the source and destination). During the second phase, the kth relay forwards a scaled version

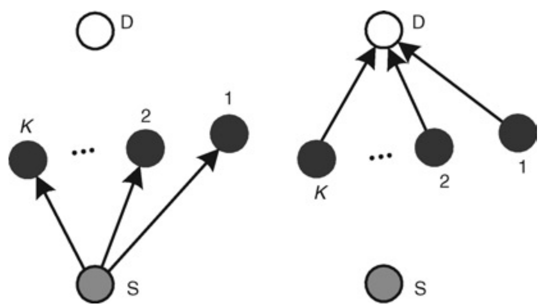

Fig. 1 *During first phase, source transmits and K AF relays listen, whereas during second phase, K AF relays amplify and re-transmit*

$$\sqrt{\frac{\mathcal{P}_k}{\mathbb{E}\{|y_k|^2\}}}\, y_k \tag{2}$$

of its received signal y_k (acquired during the first phase of the protocol). The channel is assumed to remain constant during the two phases (slow fading). This multiple AF relaying yields the outage probability as [4]:

$$\mathbb{P}_{MR-AF}(\text{outage}) = \mathbb{P}\left\{ \frac{1}{2}\log_2\left(1 + \frac{\left| \sum_{k=1}^{K} \sqrt{\frac{\mathcal{P}_k}{\Omega_{S,k}\mathcal{P}_S + N_0}}\, \alpha_{S,k}\alpha_{k,D} \right|^2}{1 + \sum_{k=1}^{K} \frac{\mathcal{P}_k |\alpha_{k,D}|^2}{\Omega_{S,k}\mathcal{P}_S + N_0}} \frac{\mathcal{P}_S}{N_0} \right) < R \right\} = \mathbb{E}_Z\left\{ 1 - \exp\left(-\frac{2^{2R}-1}{Z} \right) \right\} \tag{3}$$

where the random variable (RV) Z is given by

$$Z = \left(\sum_{k=1}^{K} \frac{\Gamma_{S,k}\Gamma_{k,D}\psi_k}{1+\Gamma_{S,k}} \right) \left(1 + \sum_{k=1}^{K} \frac{\Gamma_{k,D}\psi_k}{1+\Gamma_{S,k}} \right)^{-1} \tag{4}$$

with independent and identically distributed (IID) exponential RVs $\psi_1, \psi_2, \ldots, \psi_K$ of unit hazard rate.

Exact outage probability: To evaluate (3), we first need to find the probability density function (PDF) $p_Z(z)$ or the cumulative distribution function (CDF) $F_Z(z)$ of Z:

$$F_Z(z) = \mathbb{P}\left\{ \sum_{k=1}^{K} \frac{\Gamma_{k,D}(\Gamma_{S,k} - z)\psi_k}{1 + \Gamma_{S,k}} \le z \right\} \tag{5}$$

Without loss of generality, we assume $\Gamma_{S,1} \le \Gamma_{S,2} \le \cdots \le \Gamma_{S,K}$. Since Z and $\psi_1, \psi_2, \ldots, \psi_K$ are non-negative, $F_Z(z) = 1$ for $z \ge \Gamma_{S,K}$ and hence (3) can be written as

$$\mathbb{P}_{MR-AF}(\text{outage}) = 1 - \exp\left(\frac{2^{2R}-1}{\Gamma_{S,K}} \right) + \int_0^{\Gamma_{S,K}} F_Z(z) \exp\left(-\frac{2^{2R}-1}{z} \right) \frac{1}{z^2} dz \tag{6}$$

(a) Equal average SNRs: symmetric case:
For $\Gamma_{S,k} = \Gamma_{SR}$ and $\Gamma_{k,D} = \Gamma_{RD}$, $\forall k \in \mathcal{S}_{relay}$, the CDF $F_Z(z)$ in (6) becomes

$$F_Z(z) = \mathbb{P}\left\{ \sum_{k=1}^{K} \psi_k < \frac{(1+\Gamma_{SR})z}{\Gamma_{RD}(\Gamma_{SR} - z)} \right\} = 1 - \sum_{k=0}^{K-1} \frac{1}{k!} \left[\frac{(1+\Gamma_{SR})z}{\Gamma_{RD}(\Gamma_{SR} - z)} \right]^k \exp\left\{ -\frac{(1+\Gamma_{SR})z}{\Gamma_{RD}(\Gamma_{SR} - z)} \right\} \tag{7}$$

where the last equality follows from the fact that the sum of K IID exponential RVs has a central chi-square distribution with $2K$ degrees of freedom. Substituting (7) into (6) gives the outage probability for the symmetric multiple AF relaying.

(b) Distinct average SNRs: asymmetric case:
Consider distinct $\Gamma_{S,k}$, $\forall k \in \mathcal{S}_{relay}$, i.e. $\Gamma_{S,1} < \Gamma_{S,2} < \cdots < \Gamma_{S,K}$. Let v_k, $k = 1, 2, \ldots, K$, be

$$v_k = \frac{\Gamma_{k,D}(\Gamma_{S,k} - z)}{1 + \Gamma_{S,k}} \tag{8}$$

(i) $0 \le z < \Gamma_{S,1}$: Using (5) and the CDF of a sum of K independent exponential RVs with unequal hazard rates [5], we obtain

$$F_Z(z) = 1 - \sum_{k=1}^{K} \prod_{\substack{j=1 \\ j \ne k}}^{K} \left(1 - \frac{v_j}{v_k} \right)^{-1} e^{-z/v_k}, \quad 0 \le z < \Gamma_{S,1} \tag{9}$$

(ii) $\Gamma_{S,k} \le z < \Gamma_{S,k+1}$, $k = 1, 2, \ldots, K-1$: We can express (5) as two sums of independent exponential RVs with unequal hazard rates:

$$F_Z(z) = \mathbb{P}\left\{ \underbrace{\sum_{j=k+1}^{K} v_j\psi_j}_{\triangleq Z_1} \le z + \underbrace{\sum_{i=1}^{k} (-v_i)\psi_i}_{\triangleq Z_2} \right\} = \mathbb{E}_{Z_2}\{F_{Z_1|Z_2}(z + Z_2)\} \tag{10}$$

where $v_j > 0$ for $j = k+1, \ldots, K$ and $v_i \le 0$ for $i = 1, 2, \ldots, k$. Using again the CDF and PDF for the sum of independent and not necessarily identically distributed exponential RVs (i.e. Z_1 and Z_2) [5], (10) becomes

$$F_Z(z) = 1 - \sum_{i=1}^{k} \sum_{j=k+1}^{K} \times \left\{ \left(1 - \frac{v_i}{v_j}\right)^{-1} \prod_{\substack{p=1 \\ p \neq i}}^{k} \left(1 - \frac{v_p}{v_i}\right)^{-1} \times \prod_{\substack{q=k+1 \\ q \neq j}} \left(1 - \frac{v_p}{v_i}\right)^{-1} e^{-z/v_j} \right\} \quad (11)$$

where $\Gamma_{S,k} \leq z < \Gamma_{S,k+1}$. Therefore, the outage probability for the asymmetric multiple AF relaying can be obtained as

$$\mathbb{P}_{MR-AF}(\text{outage}) = 1 - \exp\left(-\frac{2^{2R}-1}{\Gamma_{S,K}}\right) + \sum_{k=1}^{K} \int_{\Gamma_{S,k-1}}^{\Gamma_{S,k}} F_Z(z) \exp\left(-\frac{2^{2R}-1}{z}\right) \frac{1}{z^2} dz \quad (12)$$

with $\Gamma_{S,0} = 0$ and $F_Z(z)$ given by (9) and (11).

(c) Numerical example:
We verify the analytical results by comparing with Monte-Carlo simulations. For example, we consider three AF relays ($K = 3$), $R = 0.1$ bit/s/Hz, $\mathcal{P}_S = 0.5$, $\mathcal{P}_{tot}, \mathcal{P}_k = (0.5/K)\mathcal{P}_{tot}$, and SNR $\triangleq \mathcal{P}_{tot}/N_0$ for two topologies:

- Symmetric case: $\Omega_{S,1} = \Omega_{S,2} = \Omega_{S,3} = 3$ and $\Omega_{1,D} = \Omega_{2,D} = \Omega_{3,D} = 1$.
- Asymmetric case: $\Omega_{S,1} = 1$, $\Omega_{S,2} = 2$, $\Omega_{S,3} = 3$ and $\Omega_{1,D} = \Omega_{2,D} = \Omega_{3,D} = 1$.

Fig. 2 shows the outage probability as a function of SNR and we can see that the simulation results match exactly with the analytical results.

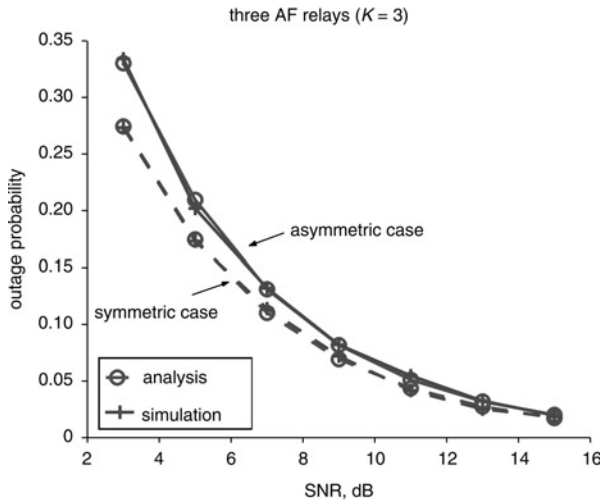

Fig. 2 *Outage probability for two (symmetric and asymmetric) topologies*

Conclusions: We have presented the exact expressions for the outage probability of multiple AF relaying in slow Rayleigh fading. Our analytical results cover general topologies and were verified through simulations. Those expressions could stimulate further research on the multiple-AF relay channel (e.g. the analysis of optimal power allocation among multiple AF relays) and enhance the fundamental understanding for the emerging and fertile area of co-operative communications.

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